Demonstration Uncertainty/Sensitivity Analysis for Modeling Hurricane Effects

Ronald L. Iman, Ph.D.
Southwest Technology Consultants
Albuquerque, NM

Prof. Mark Johnson
University of Central Florida

Prof. Tom Schroeder
University of Hawaii

FCHLPM Tallahassee, FL
October 15, 2001

Acknowledgment & Goal

- Funding provided by the Florida Commission on Hurricane Loss Projection Methodology

- Goal is to provide a demonstration uncertainty and sensitivity analysis on a simple, but realistic wind field model and damage function

- Goal is NOT to develop the next wind field model or damage function

Working Definitions

Sensitivity Analysis

- Determination of the change in response of a model to changes in model inputs and specifications

Uncertainty Analysis

- Determination of the variation or imprecision in model output resulting from the collective variation in the model inputs

Overview

- Principal reason for performing an uncertainty analysis and sensitivity is to provide the modelers with new insights into their models
- Identify major contributors to the magnitude of the output of the different modules
- Identify major contributors to the uncertainty in the output of the different modules
- Recognize changes in the importance ranking of these contributors over space and time

Overview

- Demonstration analysis provides guidance on how such information can be obtained and communicated
- Such information leads to:
  - Output that can be compared to wind engineering experience
  - Models that can be rigorously defended
- Quality control
  - Experience has shown that "properly" selected sample input characteristics will likely find mistaken program logic or inconsistencies

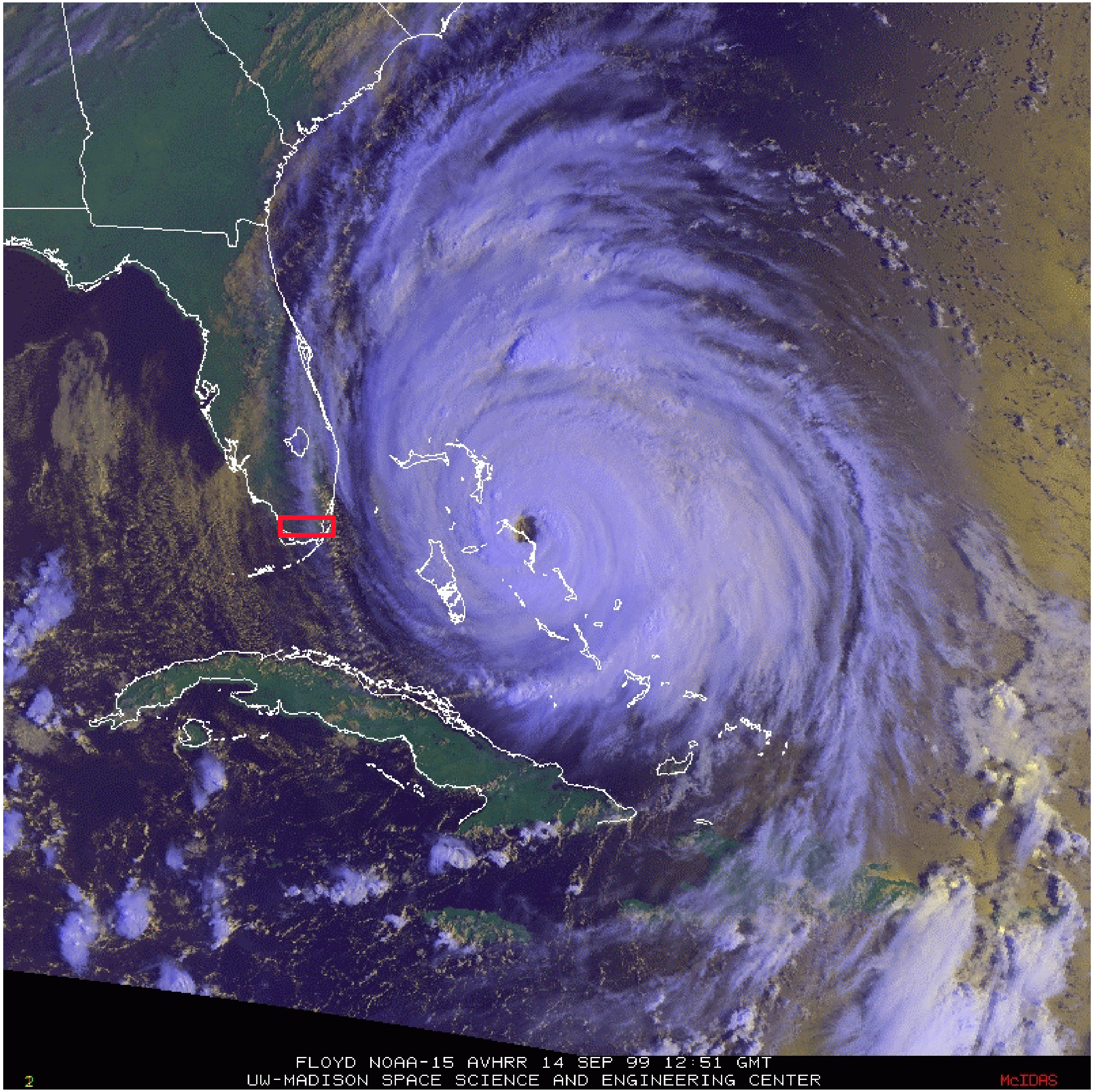
Overview

- Model-to-model comparisons
  - Identify major contributors to loss cost and help explain why the models produce different results

- Extend methodology to other model parameters
  - Once the UA/SA concepts are understood, the methodology can be easily extended to other model parameters (e.g. FFP added to CP, Rmax and V_T mix)

Outline of Presentation

- Surrogate wind speed model
- Sensitivity analysis results for wind speed
- Sensitivity analysis for loss cost
- Uncertainty analysis results for wind speed
- Uncertainty analysis for loss cost


FLOYD NOAA-15 AVHRR 14 SEP 99 12:51 GMT
UW-MADISON SPACE SCIENCE AND ENGINEERING CENTER
McIDAS

Floyd

Hurricane Categories

Saffir-Simpson Scale

Category	Speed (mph)	CP (mB)	Damage
1	74-95	= 980	Minimal
2	96-110	965-979	Moderate
3	111-130	945-964	Extensive
4	131-155	920-944	Extreme
5	> 155	< 920	Catastrophic

Hurricane Parameters Used in Demonstration

Central Pressure (CP)

- Barometric pressure at the center of the storm
- Average sea level pressure is 1013mB
- The lower CP, the stronger the storm in terms of wind speed
- Lowest pressure ever measured in a hurricane in the Atlantic basin was 888mB (Hurricane Gilbert)

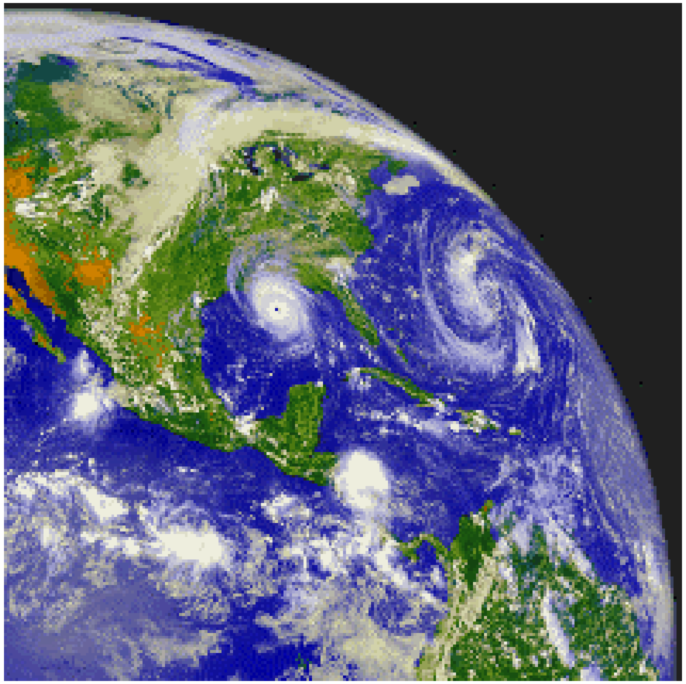
Hurricane Parameters Used in Demonstration

Radius of Maximum Winds (Rmax)

- Radius (mi) from storm center to the point of maximum winds (the eye wall) surrounding a storm

- Varies with intensity of storm

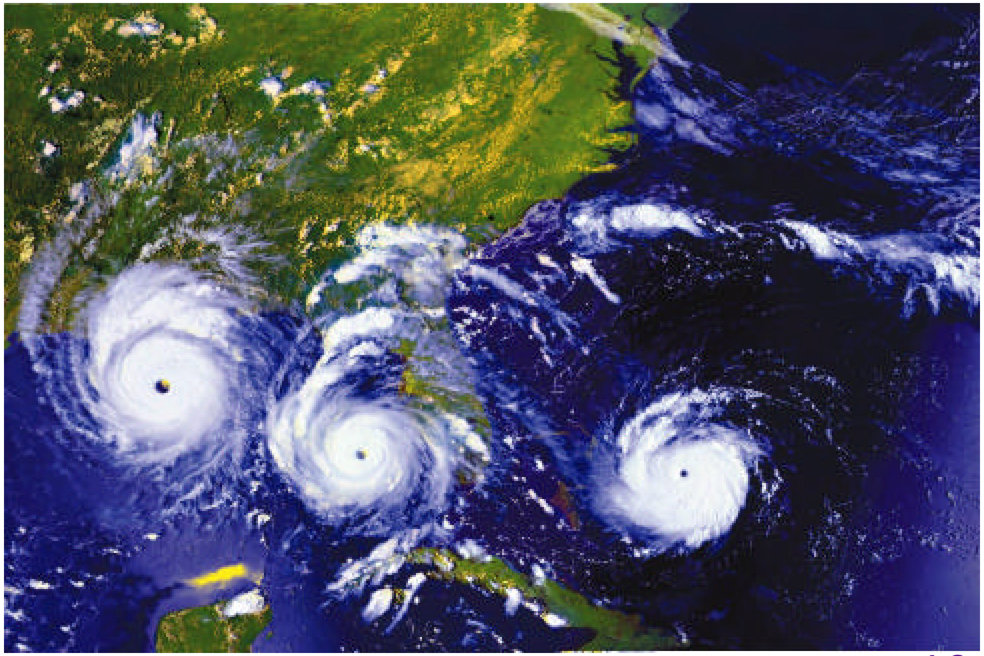
Andrew



Hurricane Parameters Used in Demonstration

Forward Speed (V_T)

- Speed (mph) at which the storm is moving along the earth's surface (not the speed at which winds are circulating around the storm center)
- Slow: 3mph
- Average: 10 to 15mph
- Fast: 20-30mph

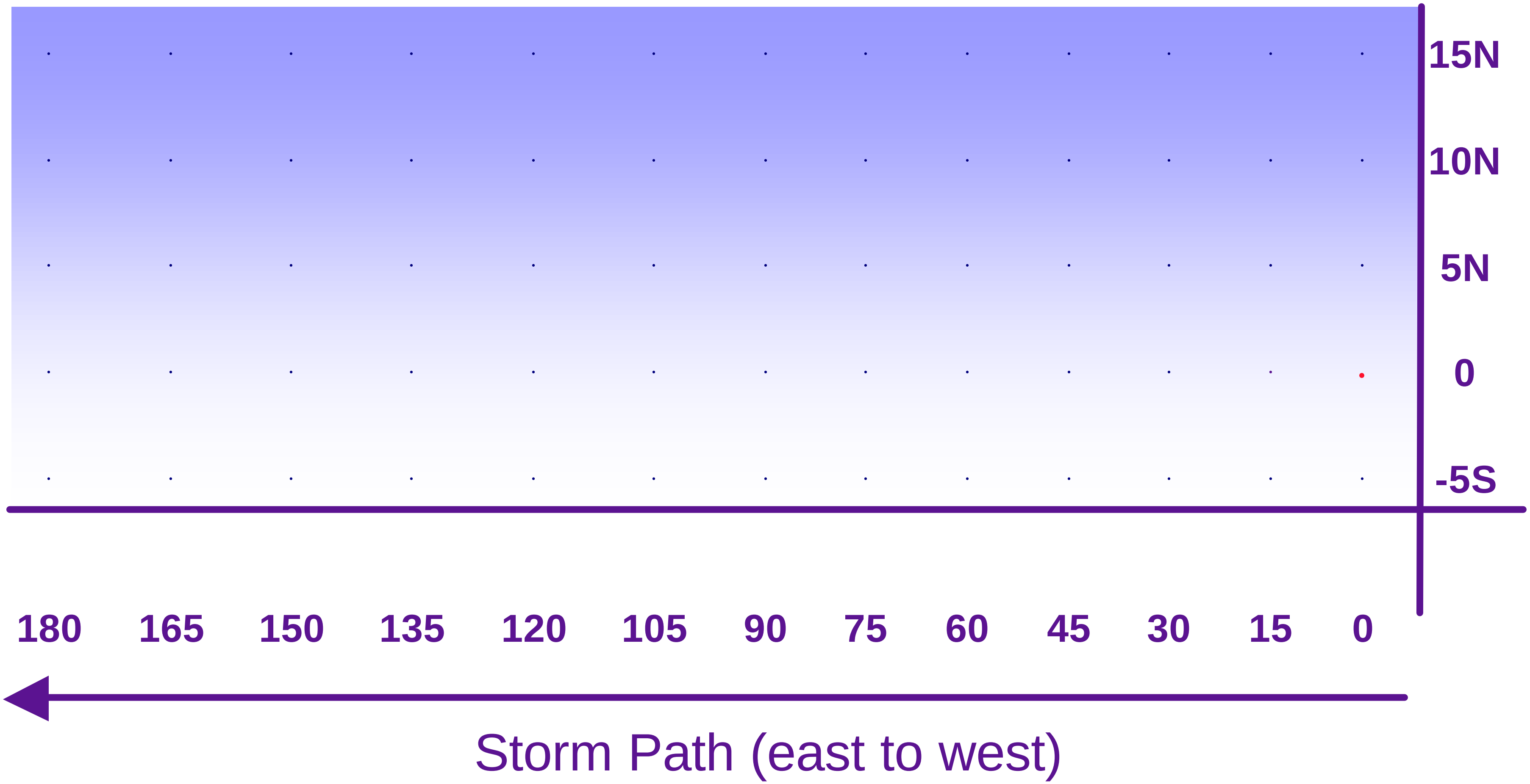


Hurricane Parameters Used in Demonstration

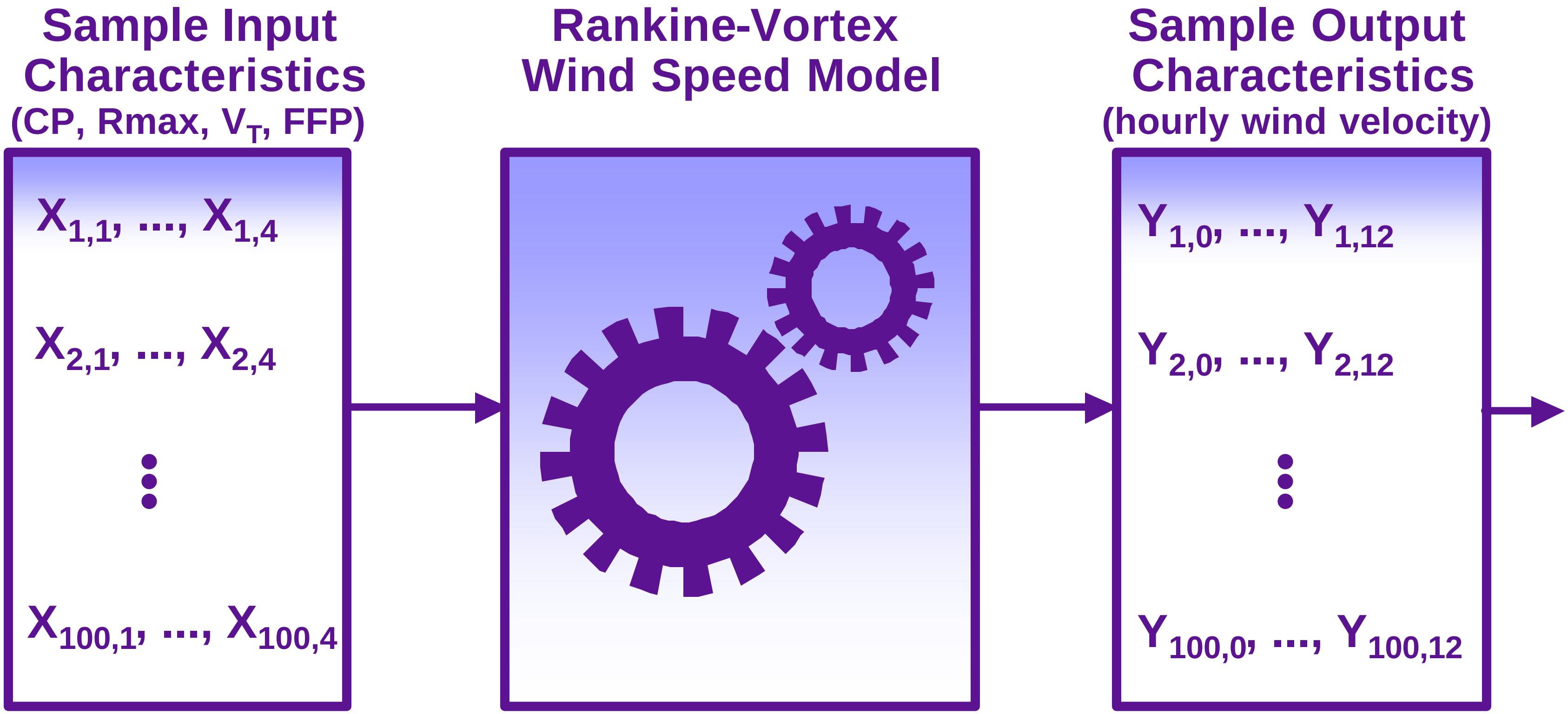
Far Field Pressure (FFP)

- Background atmospheric pressure, nominally at the periphery of the actual storm circulation
- Average sea level pressure is 1013mB
- The higher FFP is relative to CP, the stronger the storm and hence, the higher the wind speed

Tracking Grid



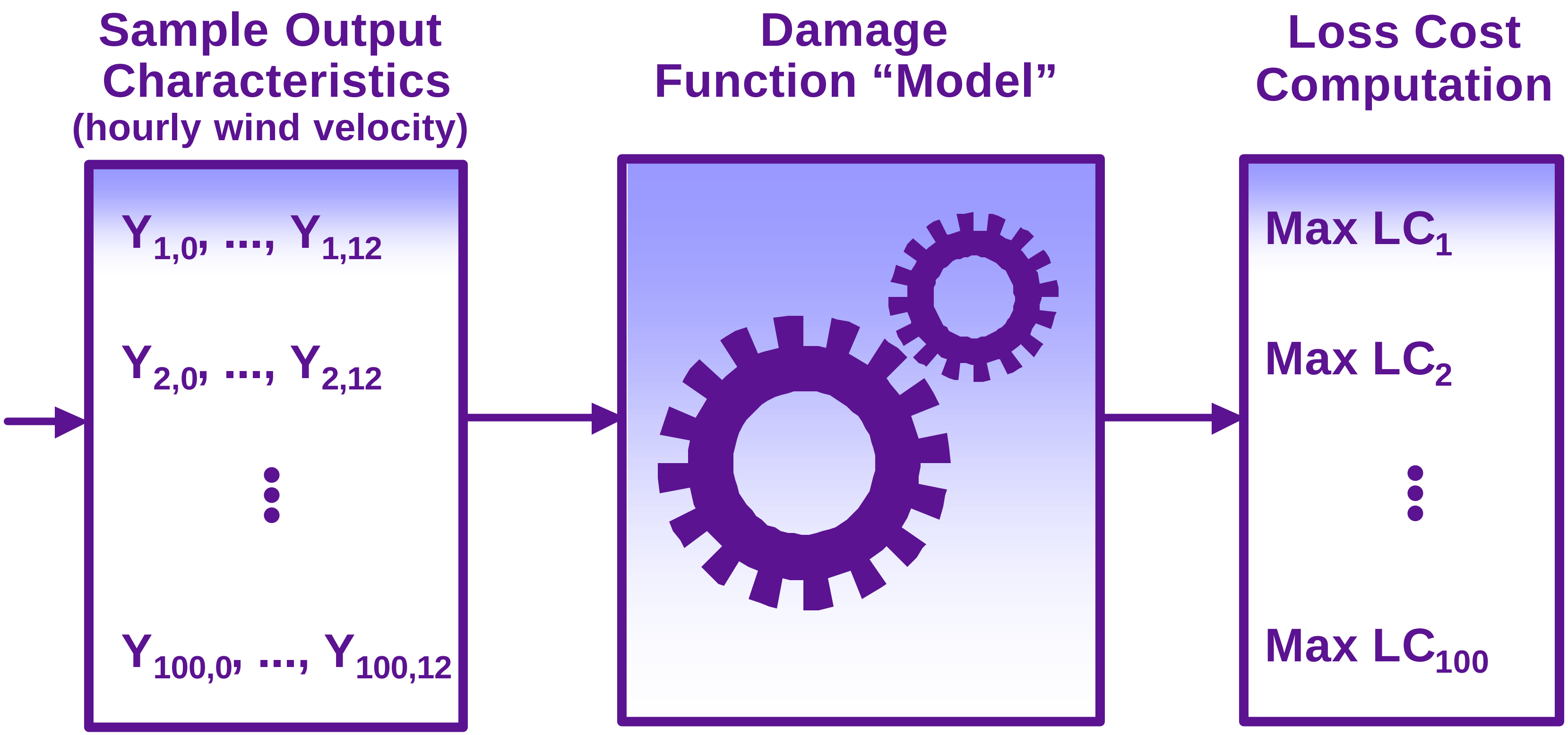
- V_T = 10 mph moves the eye to (120, 0) in 12hr
- V_T = 15 mph moves the eye to (180, 0) in 12hr
- V_T = 20 mph moves the eye to (240, 0) in 12hr



Step 1. Generate samples for input characteristics (Latin Hypercube Sampling with n=100)

Step 2. Run computer wind speed model using matrix of LHS input

Step 3. Wind speeds are recorded hourly for each grid point for each LHS input vector (5 x 13 x 13 x 100 = 84,500 wind speeds for each storm Category)

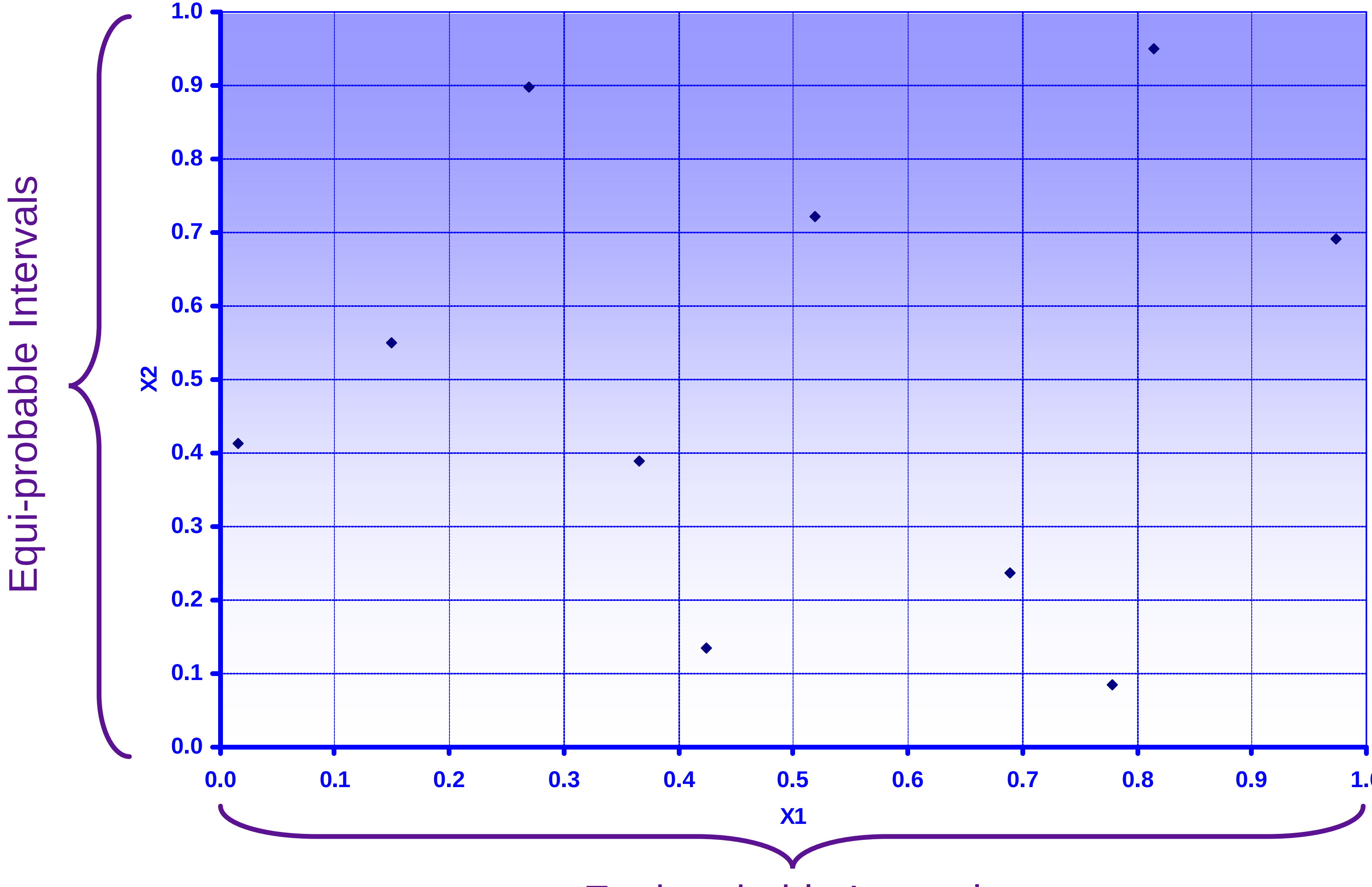


Step 3. Wind speeds are recorded hourly for each grid point for each LHS input vector

Step 4. Compute damage for wind speed using simple cubic function for demonstration purposes

Step 5. Convert damage to loss cost, use max loss cost over time at each vertex and sum over entire grid to get total loss cost (100 values)

LHS for Two Variables with *n* = 10



Uncertainty and Sensitivity Analysis

Sensitivity Analysis

- Determine which X’s influence the magnitude of Y at time t

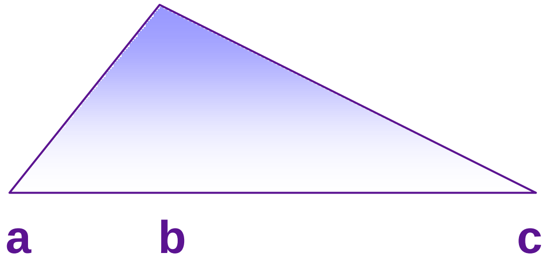
Uncertainty Analysis

- Determine which X’s contribute to the uncertainty in Y at time t

Ranges of Uncertainty for Input Parameters

	Category 1	Category 5
CP	980mB = CP = 990mB	$900\text{mB} \leq \text{CP} \leq 920\text{mB}$
Rmax	$12\text{mi} \leq \text{Rmax} \leq 21\text{mi}$	$6\text{mi} \leq \text{Rmax} \leq 12\text{mi}$
V_T	$10\text{mph} \leq V_T \leq 20\text{mph}$	$10\text{mph} \leq V_T \leq 20\text{mph}$
FFP	$1010\text{mB} \leq \text{FFP} \leq 1016\text{mB}$	$1010\text{mB} \leq \text{FFP} \leq 1016\text{mB}$

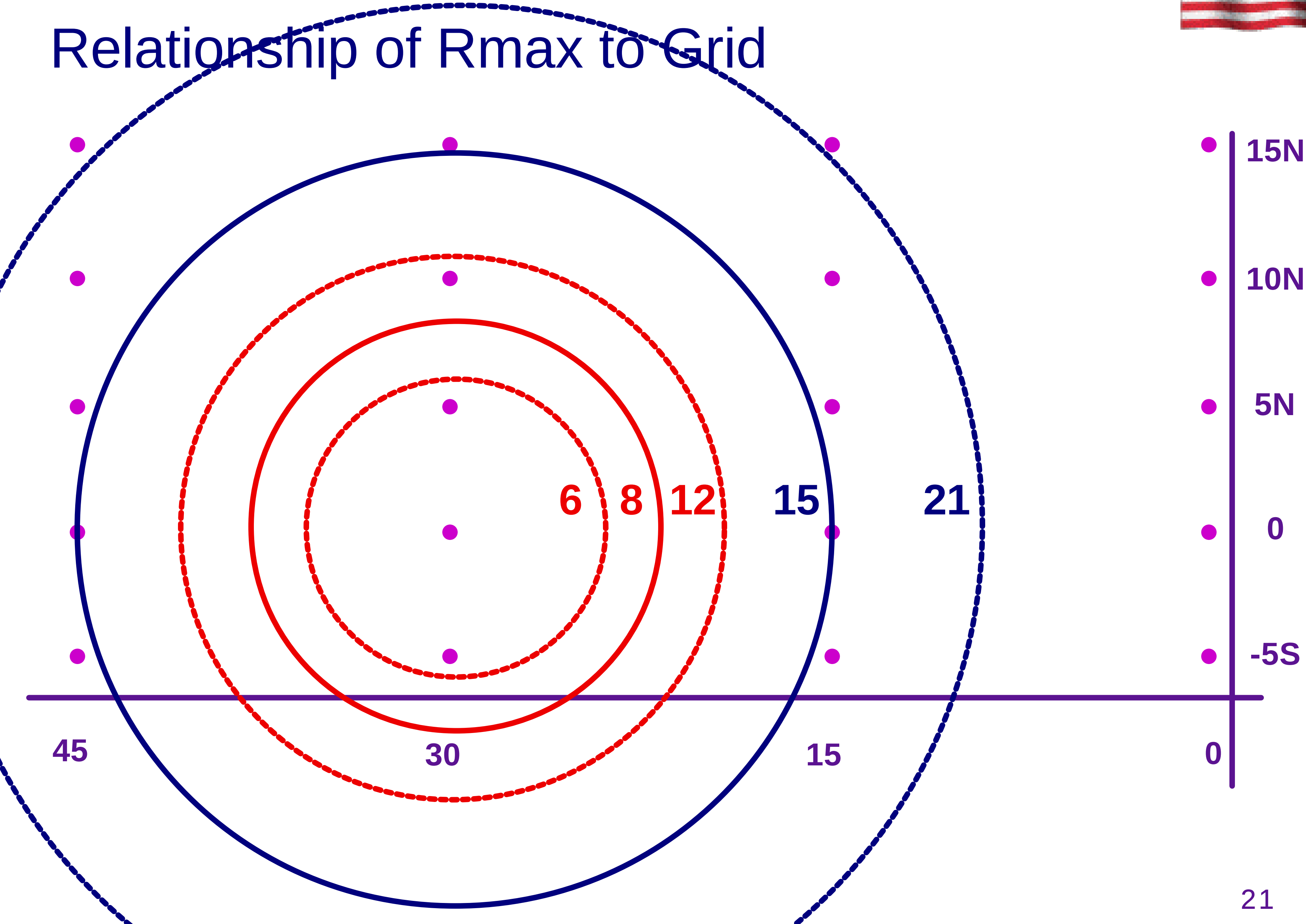
Triangular Probability Distributions for Input Parameters



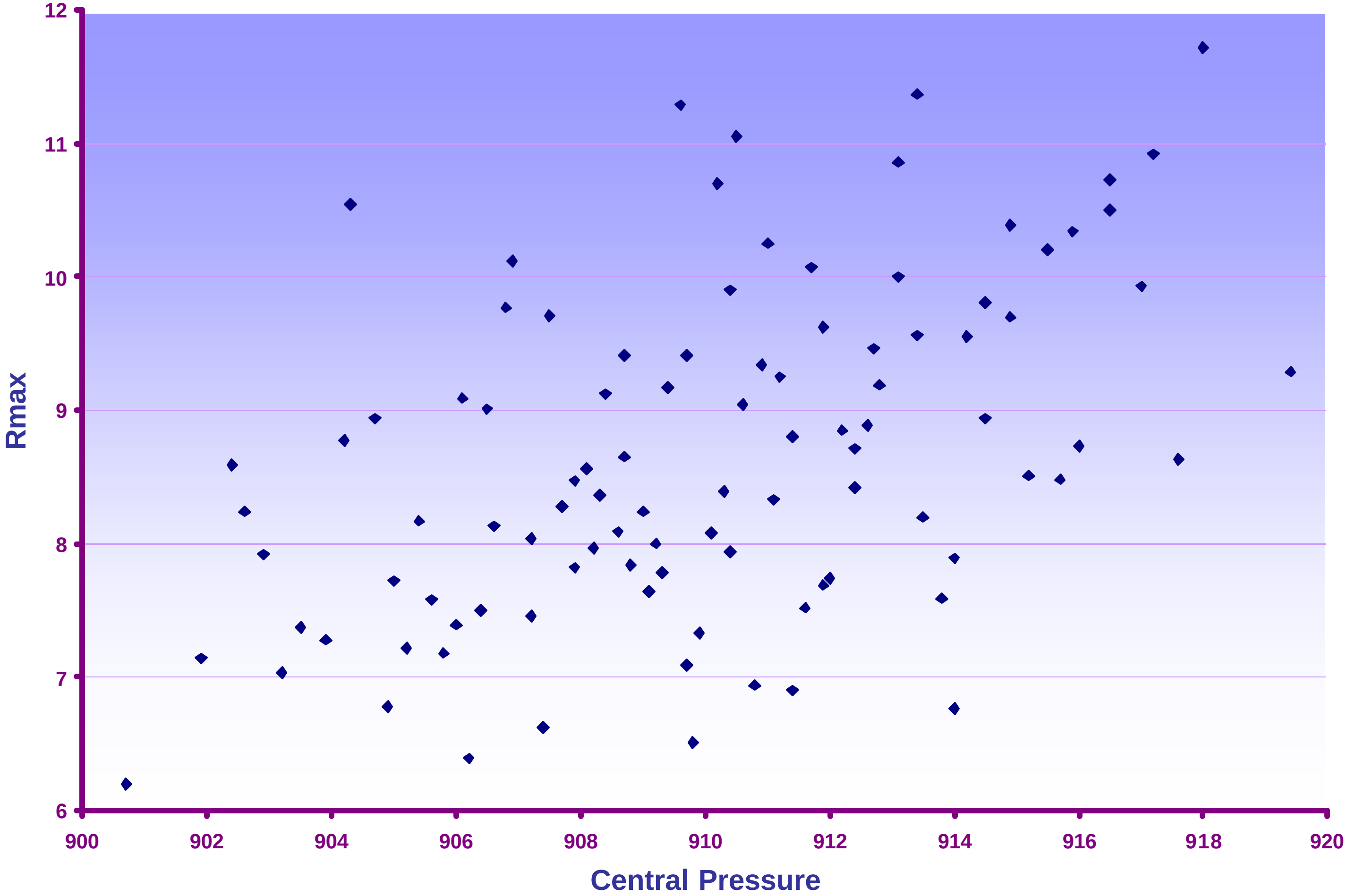
Latin Hypercube Sampling with n = 100

		Category 1			Category 5	
		Mean	St Dev		Mean	St Dev
CP	a=980 b=985 c=990	985	2.04	a=900 b=910 c=920	910	4.08
Rmax	a=12 b=15 c=21	16	1.87	a=6 b=8 c=12	8.67	1.25
V_T	a=10 b=15 c=20	15	2.04	a=10 b=15 c=20	15	2.04
FFP	a=1010 b=1013 c=1016	1013	1.22	a=1010 b=1013 c=1016	1013	1.22

Relationship of Rmax to Grid

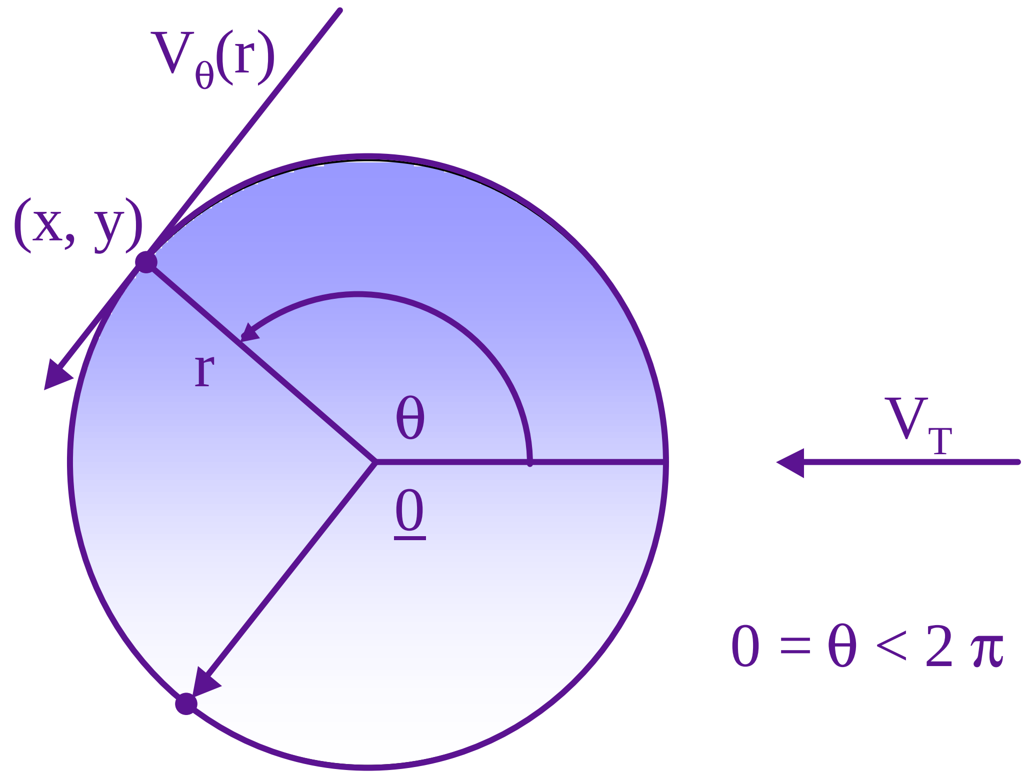


Correlation of Cp and Rmax for Category 5



Correlation: 0.5 for Category 5 and 0.25 for Category 1

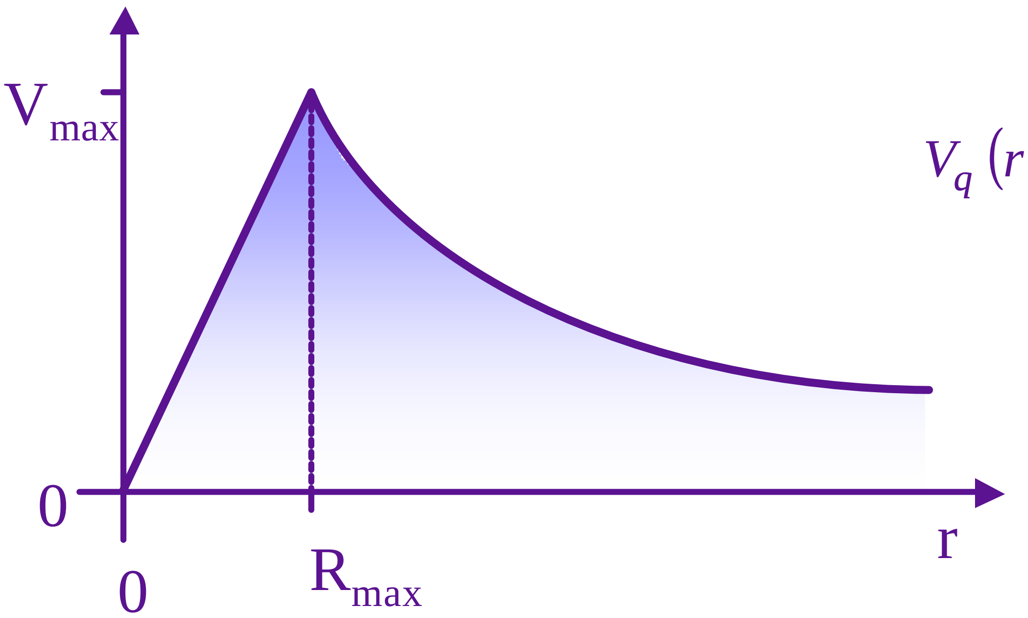
Tangential Wind Velocity Vector



$(r \cos(\theta + \pi/2), r \sin(\theta + \pi/2))$

Rankine-vortex Function for Wind Speed

$$V_{\max} = 14 * 1.15\sqrt{FFP - CP} - V_T$$



$$V_q(r) = \frac{V_{\max}}{R_{\max}} r \qquad 0 \le r \le R_{\max}$$

$$= V_{\max}\left(\frac{R_{\max}}{r}\right)^{0.6} \qquad r \ge R_{\max}$$

Surrogate Decay:
Category 1: increase CP by 1 mB/hr
Category 5: increase CP by 3 mB/hr

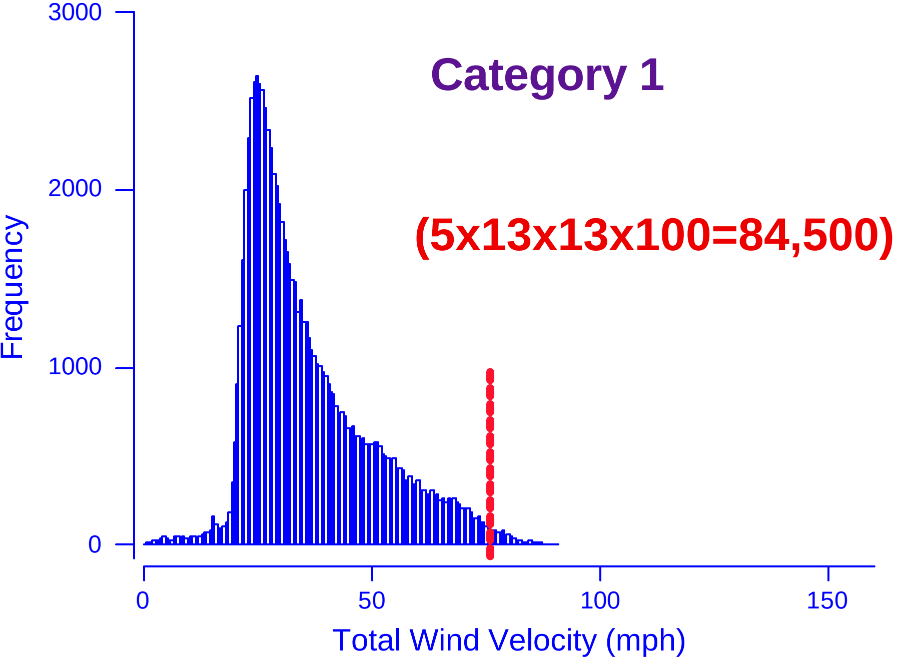
Total Wind Speed

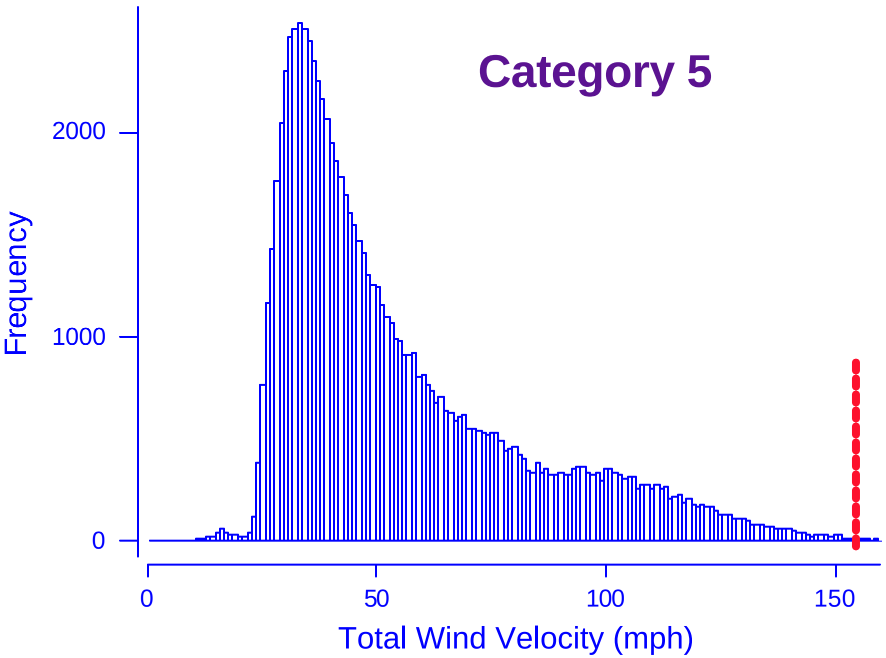
$$V_{Total}(x,y)=\sqrt{\left[V_q(r)\cos\left(q+\frac{p}{2}\right)-V_T\right]^2+\left[V_q(r)\sin\left(q+\frac{p}{2}\right)\right]^2}$$

Sample Input Characteristics and V_{Total} for Category 5 at (30,0) for t = 1hr

Sample	CP	Rmax	V_T	FFP	V_{Total}
1	911.7	10.07	14.16	1013.4	112.0
2	909.1	7.64	18.15	1012.3	111.4
3	909.0	8.24	16.85	1012.3	110.4
…	…	…	…	…	…
100	907.9	7.82	14.42	1012.9	99.0

Distribution of Wind Speeds by Category



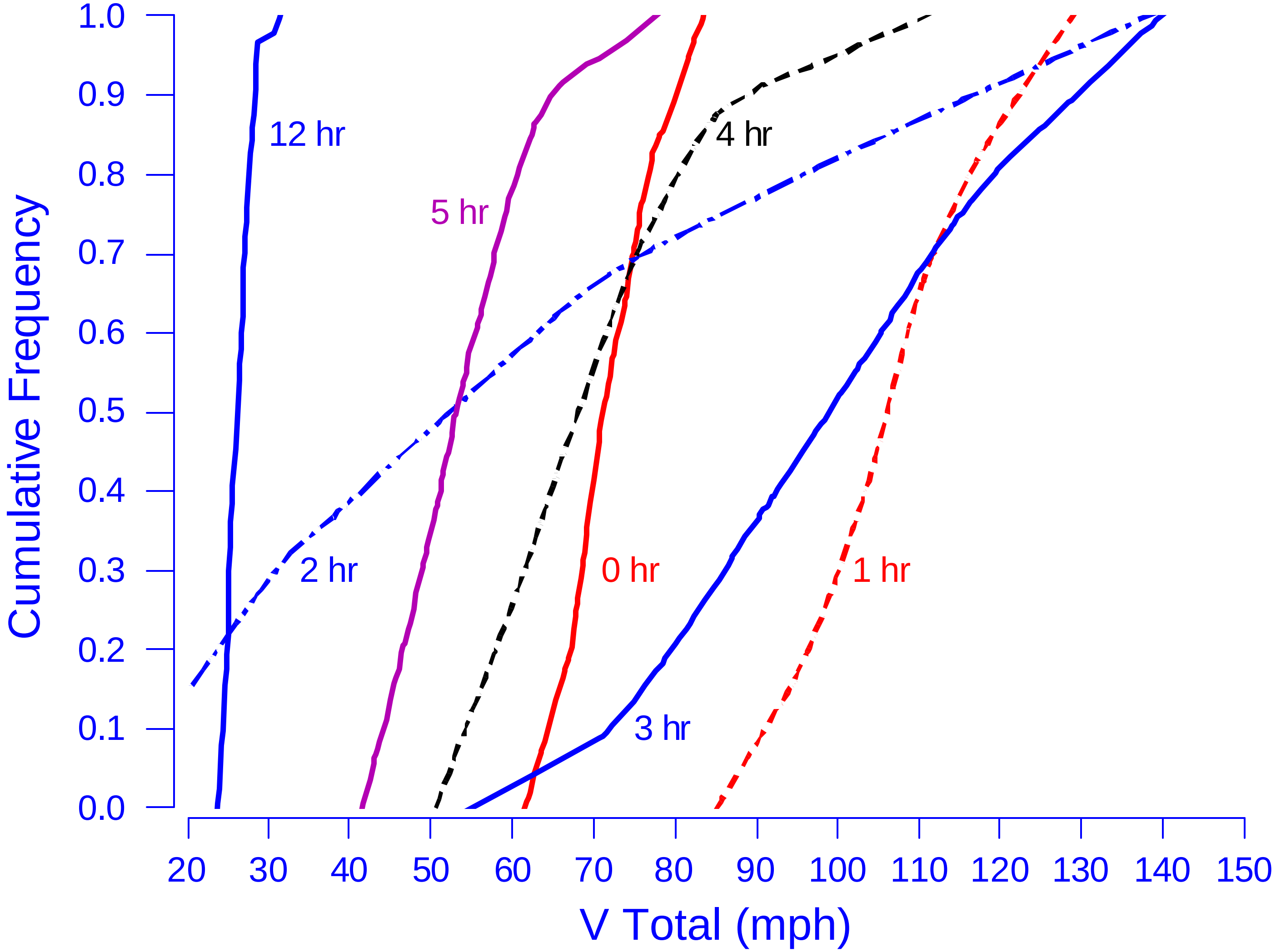


	Category 1	Category 5
Mean	**36.0**	**54.9**
St. dev.	**13.9**	**26.8**
Median	**31.8**	**45.6**
Minimum	**0.1**	**10.3**
Maximum	**91.1**	**164.5**

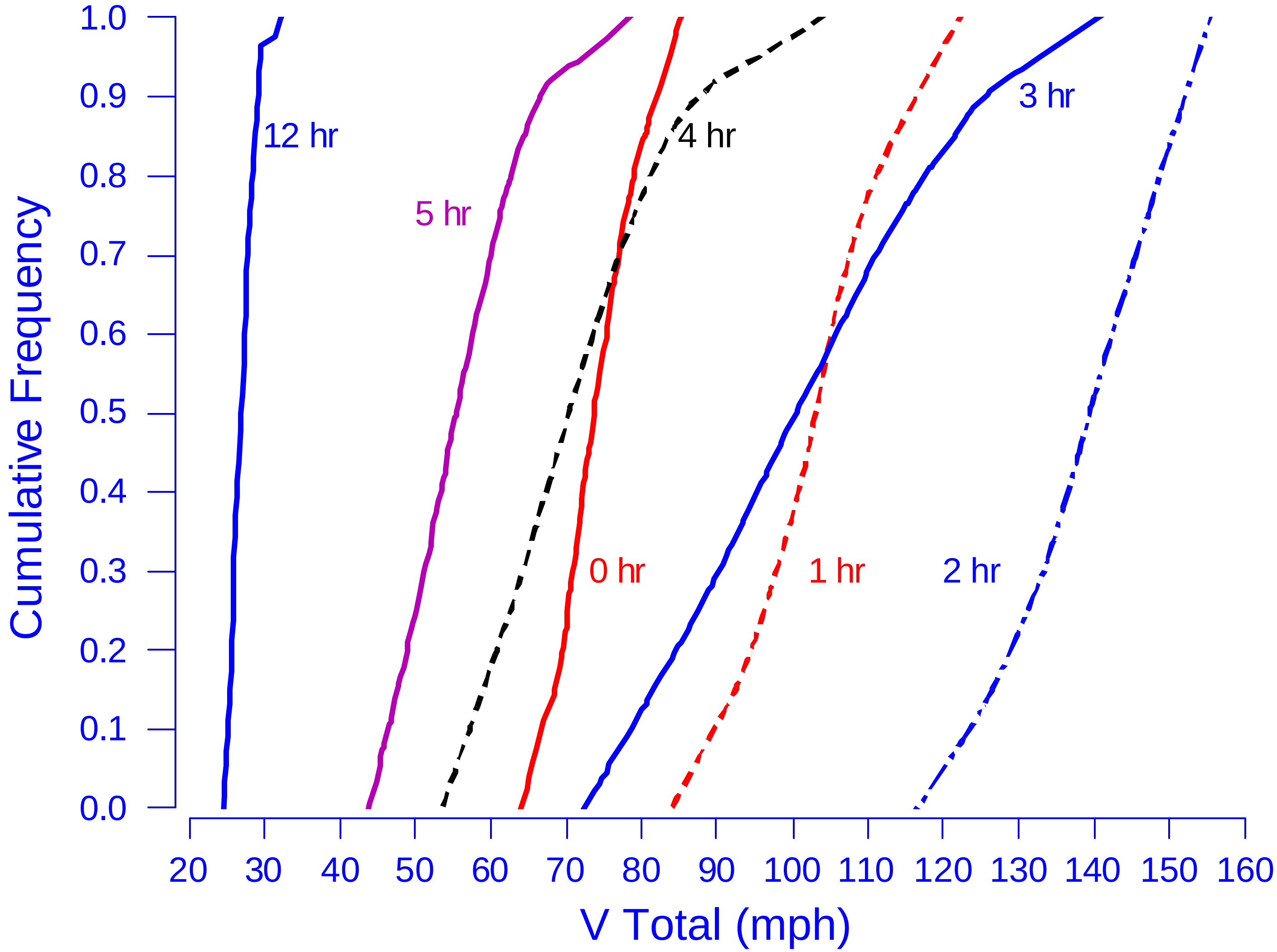
Frequency of Max Wind Speeds Per Input Vector

Category 1		Category 5	
Max Wind Speed (mph)	Frequency	Max Wind Speed (mph)	Frequency
WS < 74	8	135 = WS < 140	3
74 = WS < 80	36	140 = WS < 145	14
80 = WS < 85	37	145 = WS < 150	29
85 = WS < 90	17	150 = WS < 155	24
90 = WS < 95	2	WS = 155	30
	100		100

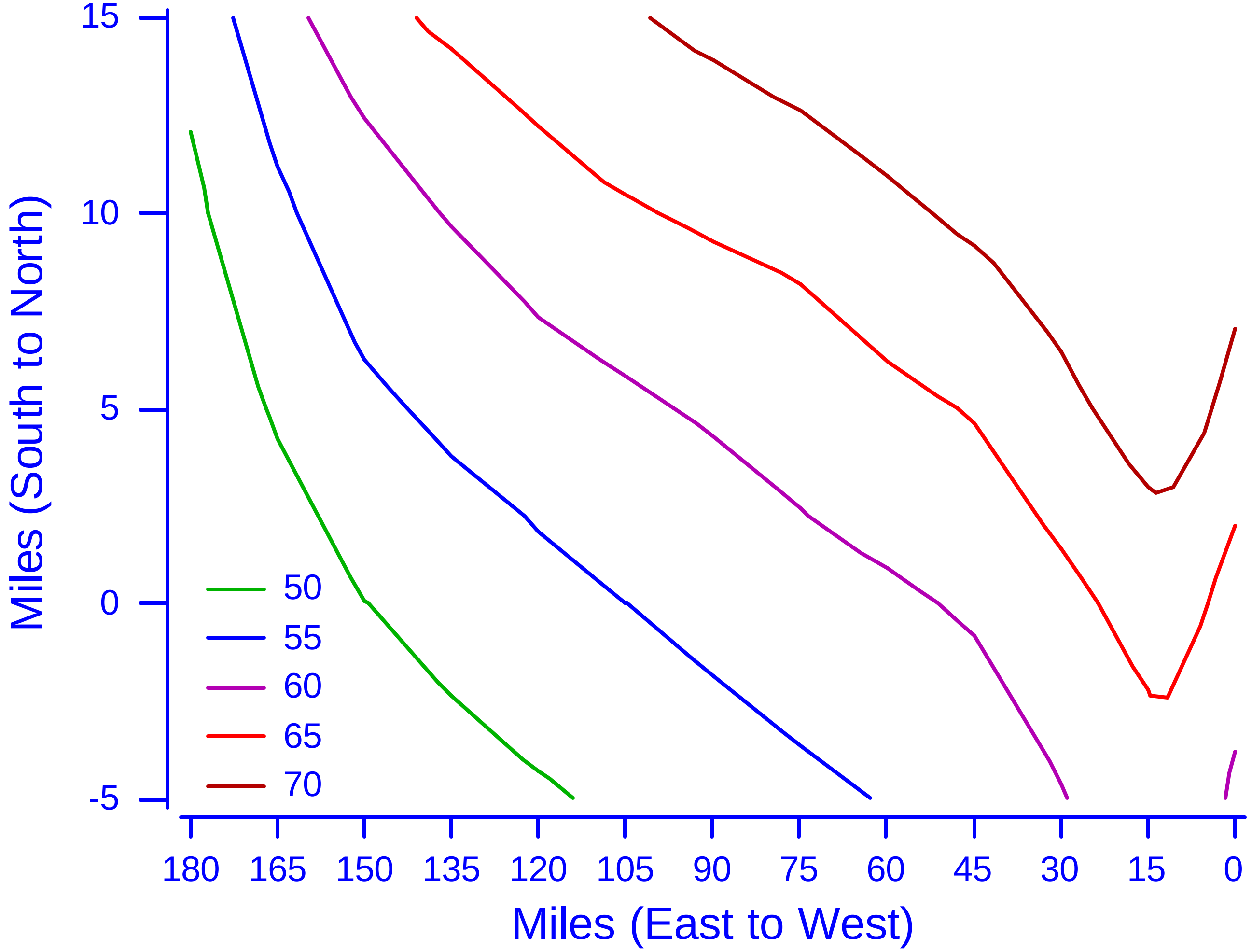
Hourly Distributions of Category 5 Wind Speeds at (30, 0)



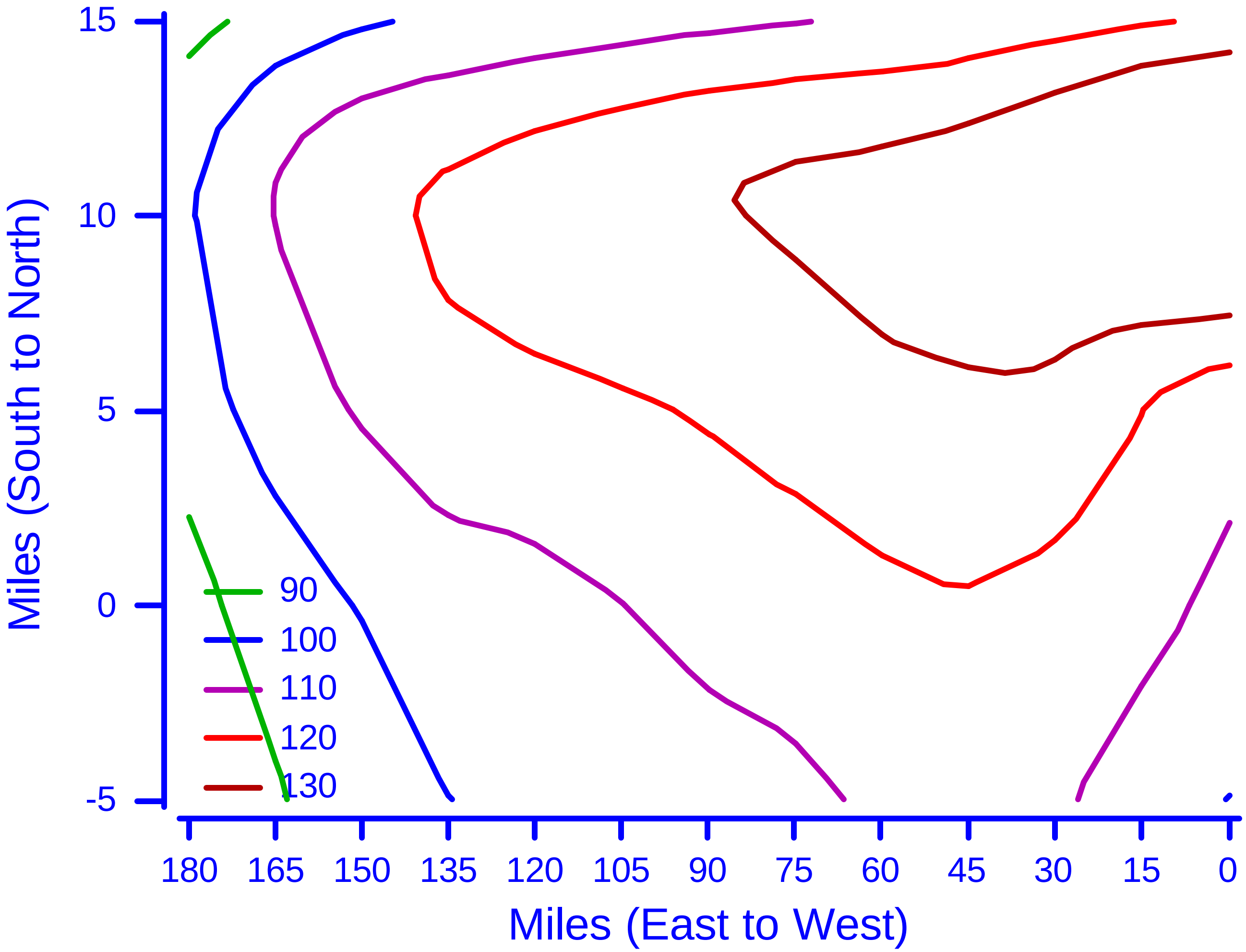
Hourly Distributions of Category 5 Wind Speeds at (30, 10)



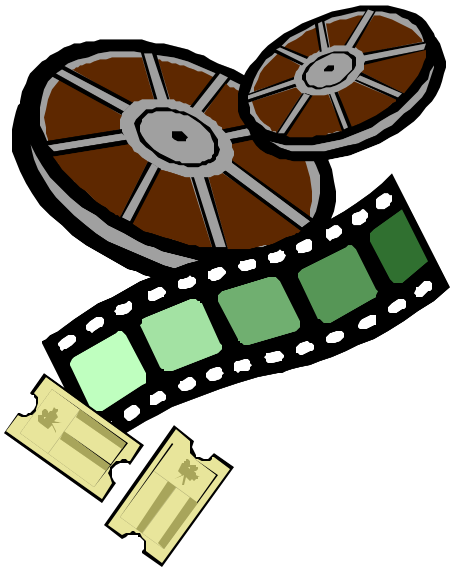
Contours of Average Max Winds for Category 1



Contours of Average Max Winds for Category 5



Movies for Categories 1 and 5: Wind Speed Tracks

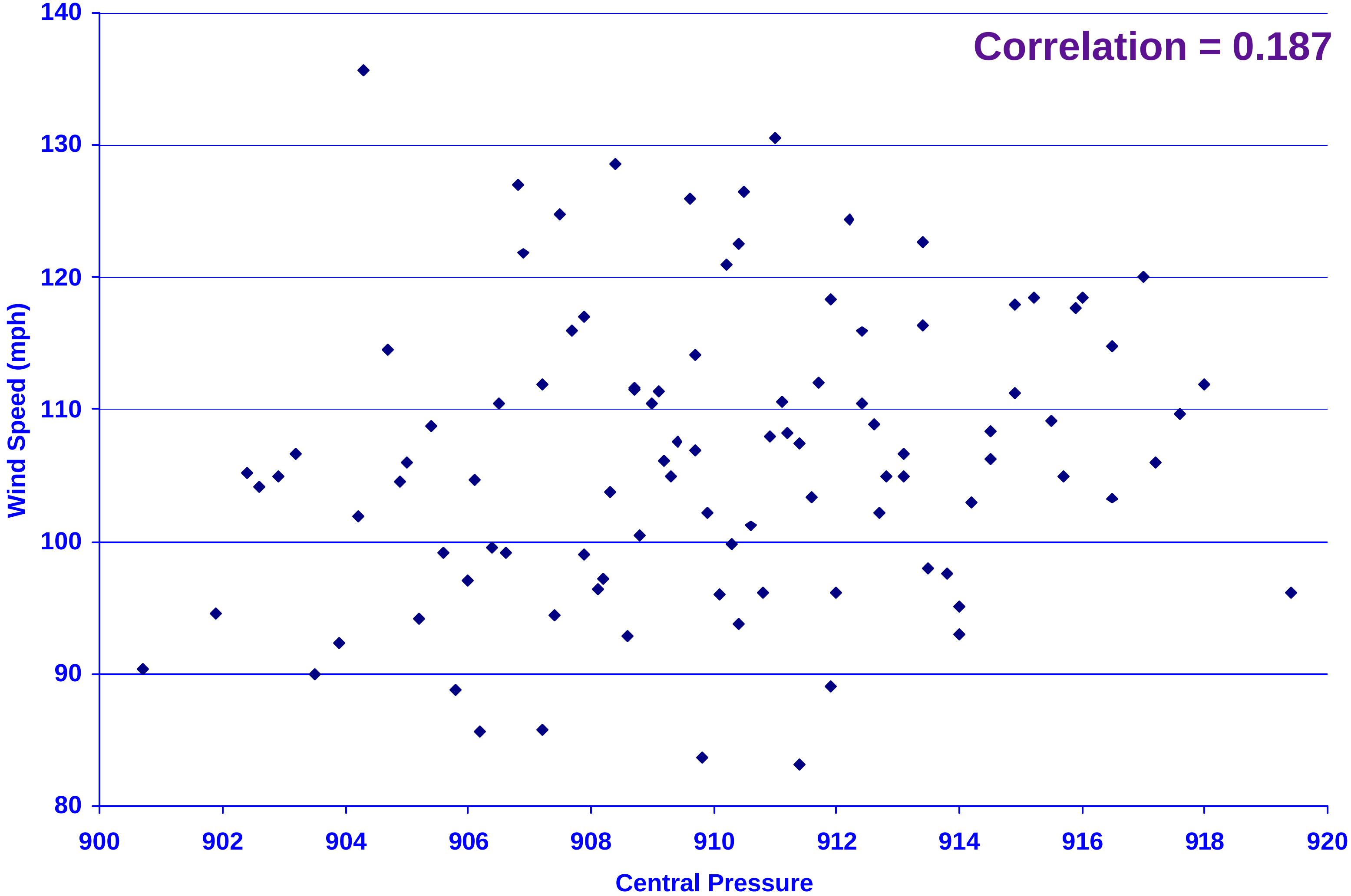


Sensitivity Analysis: Which X's Influence V_{Total}

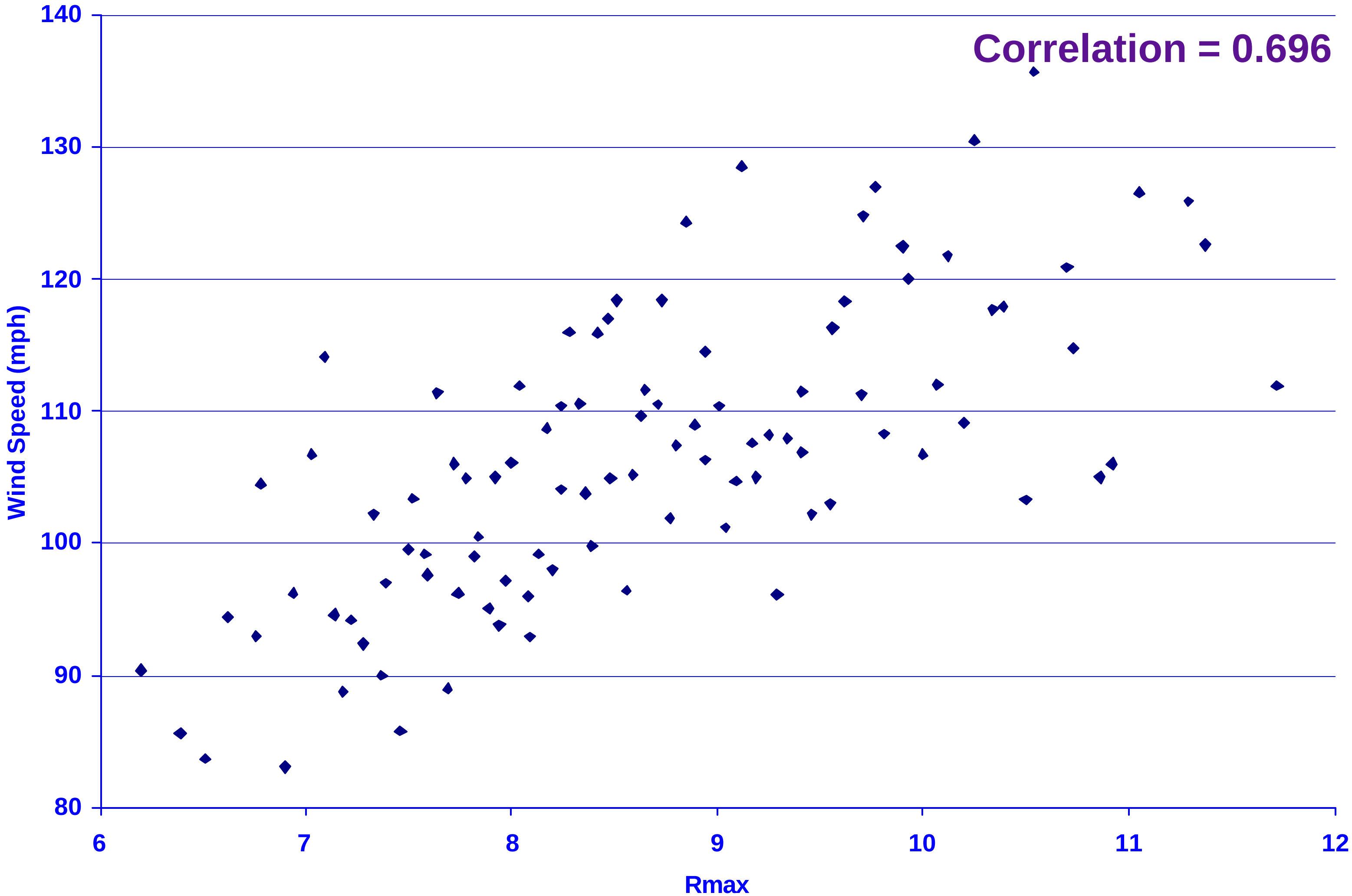
Cat 5 Simple Correlation Matrix at (30, 0) for t=1hr

CP	**1.000**				
Rmax	**0.490**	**1.000**			
V_T	**0.005**	**-0.012**	**1.000**		
FFP	**-0.012**	**-0.009**	**0.006**	**1.000**	
V_{Total}	**0.187**	**0.696**	**0.675**	**0.066**	**1.000**
	CP	**Rmax**	**V_T**	**FFP**	**V_{Total}**

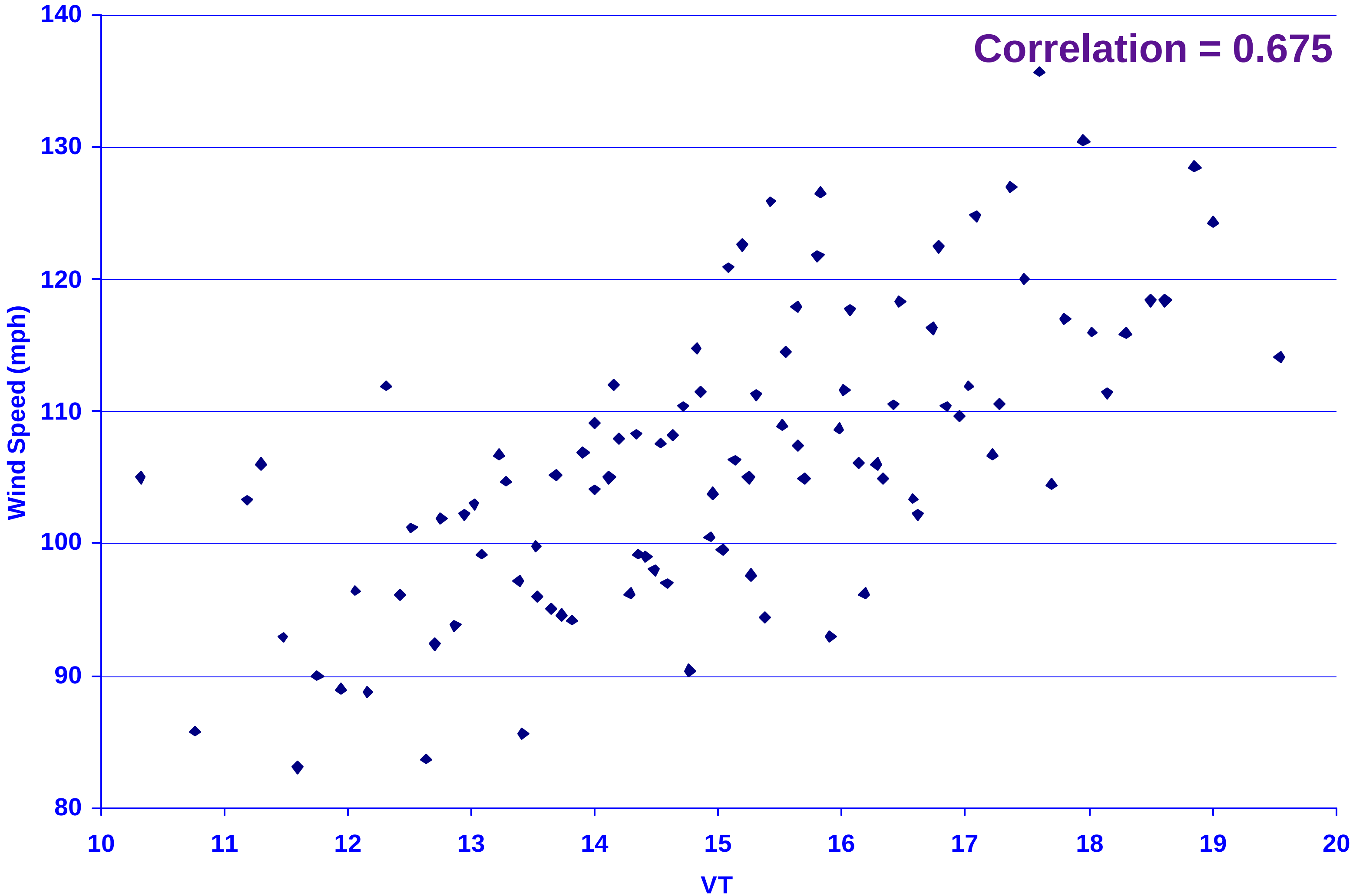
Scatterplot of V_{Total} vs CP



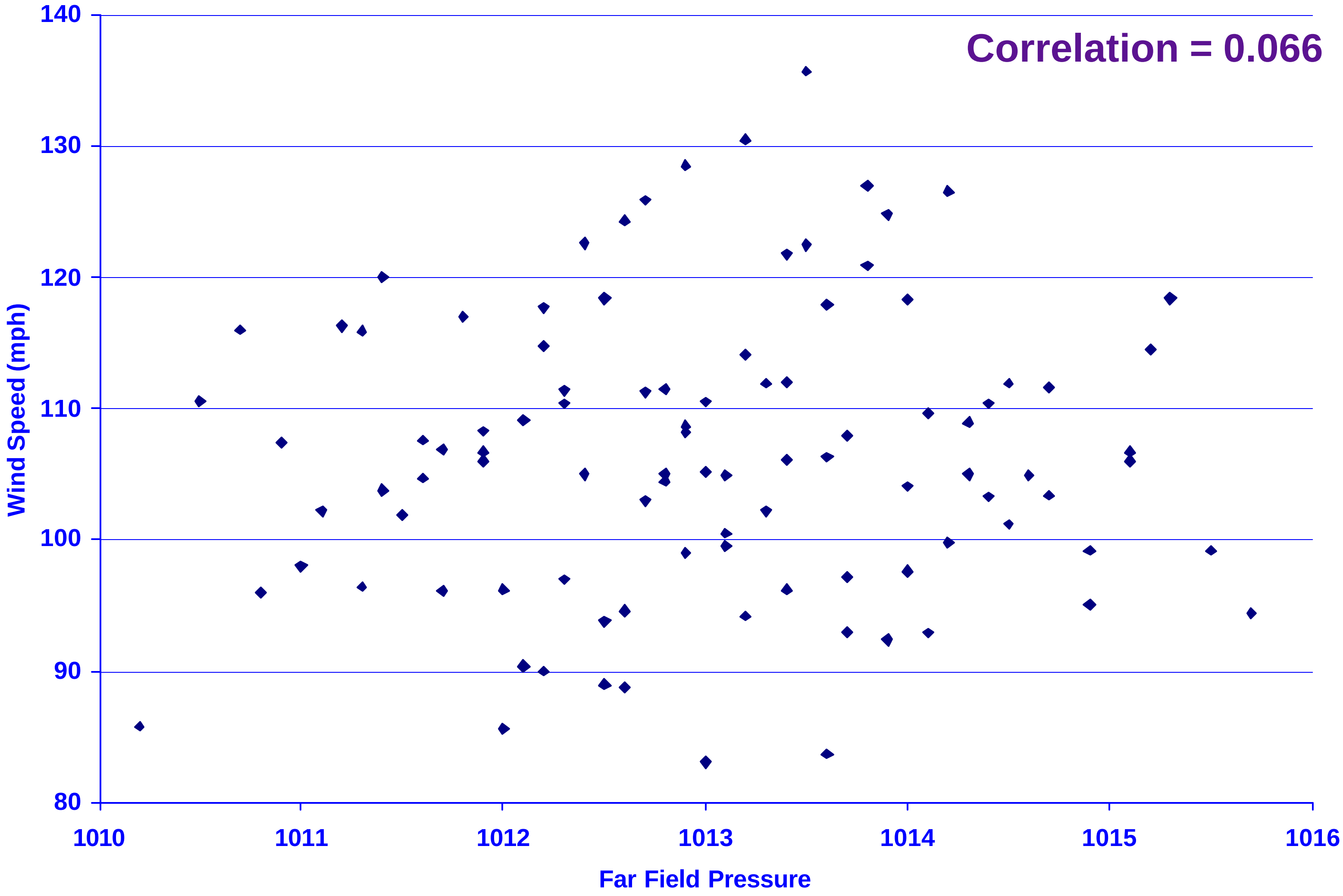
Scatterplot of V_{Total} vs Rmax



Scatterplot of V_{Total} vs V_T



Scatterplot of V_{Total} vs FFP



Sensitivity Analysis for V_{Total}

Partial Correlation Coefficients (PCC)

- Measures the degree of linear relationship between a given input parameter and V_{Total} following an adjustment to remove the linear effect of the other three input parameters

Standardized Regression Coefficients (SRC)

- Permits comparison of the regression coefficients in common or standardized units

Inverse of Correlation Matrix

$1/(1-R_{x_1}^2)$				
c_{21}	$1/(1-R_{x_2}^2)$			
c_{31}	c_{32}	$1/(1-R_{x_3}^2)$		
c_{41}	c_{42}	c_{43}	$1/(1-R_{x_4}^2)$	
$-b_1/(1-R_y^2)$	$-b_2/(1-R_y^2)$	$-b_3/(1-R_y^2)$	$-b_4/(1-R_y^2)$	$1/(1-R_y^2)$

Partial Correlation Coefficient (PCC)

$$PCC_{x_j y} = -c_{jy} \Big/ \sqrt{c_{jj} c_{yy}}$$

$$\begin{bmatrix}
1/\left(1-R_{x_1}^2\right) & & & & \\
c_{21} & 1/\left(1-R_{x_2}^2\right) & & & \\
c_{31} & c_{32} & 1/\left(1-R_{x_3}^2\right) & & \\
c_{41} & c_{42} & c_{43} & 1/\left(1-R_{x_4}^2\right) & \\
-b_1/\left(1-R_y^2\right) & -b_2/\left(1-R_y^2\right) & -b_3/\left(1-R_y^2\right) & -b_4/\left(1-R_y^2\right) & 1/\left(1-R_y^2\right)
\end{bmatrix}$$

Standardized Regression Coefficient (SRC)

$$Y = \beta_0 + \beta_1 \, CP + \beta_2 \, Rmax + \beta_3 \, V_T + \beta_4 \, FFP$$

mB **mi** **mph** **mB**

Standardize each input variable

$$Y^* = b_0 + b_1 \, CP^* + b_2 \, Rmax^* + b_3 \, V_T^* + b_4 \, FFP^*$$

Standardized Regression Coefficient (SRC)

$$SRC_{x_j y} = b_j$$

$$\begin{bmatrix}
1/\left(1-R^2_{x_1}\right) & & & & \\
c_{21} & 1/\left(1-R^2_{x_2}\right) & & & \\
c_{31} & c_{32} & 1/\left(1-R^2_{x_3}\right) & & \\
c_{41} & c_{42} & c_{43} & 1/\left(1-R^2_{x_4}\right) & \\
-b_1/\left(1-R^2_y\right) & -b_2/\left(1-R^2_y\right) & -b_3/\left(1-R^2_y\right) & -b_4/\left(1-R^2_y\right) & 1/\left(1-R^2_y\right)
\end{bmatrix}$$

Relationship between SRC and PCC

$$PCC_{x_j y} = \frac{-(-b_j)/(1-R_y^2)}{\sqrt{\left(\frac{1}{1-R_{x_j}^2}\right)\left(\frac{1}{1-R_y^2}\right)}}$$

$$= b_j \sqrt{\frac{1-R_{x_j}^2}{1-R_y^2}}$$

Cat 5 Correlation Matrix and Inverse at (30, 0) for t=1hr

Correlation Matrix

	CP	Rmax	V_T	FFP	V_{Total}
CP	1.000				
Rmax	0.490	1.000			
V_T	0.005	-0.012	1.000		
FFP	-0.012	-0.009	0.006	1.000	
V_{Total}	0.187	0.696	0.675	0.066	1.000

Inverse Matrix

	CP	Rmax	V_T	FFP	V_{Total}
CP	5.421				
Rmax	-16.490	61.000			
V_T	-13.324	50.856	44.152		
FFP	-0.976	3.793	3.230	1.243	
V_{Total}	19.411	-74.151	-62.937	-4.721	91.000

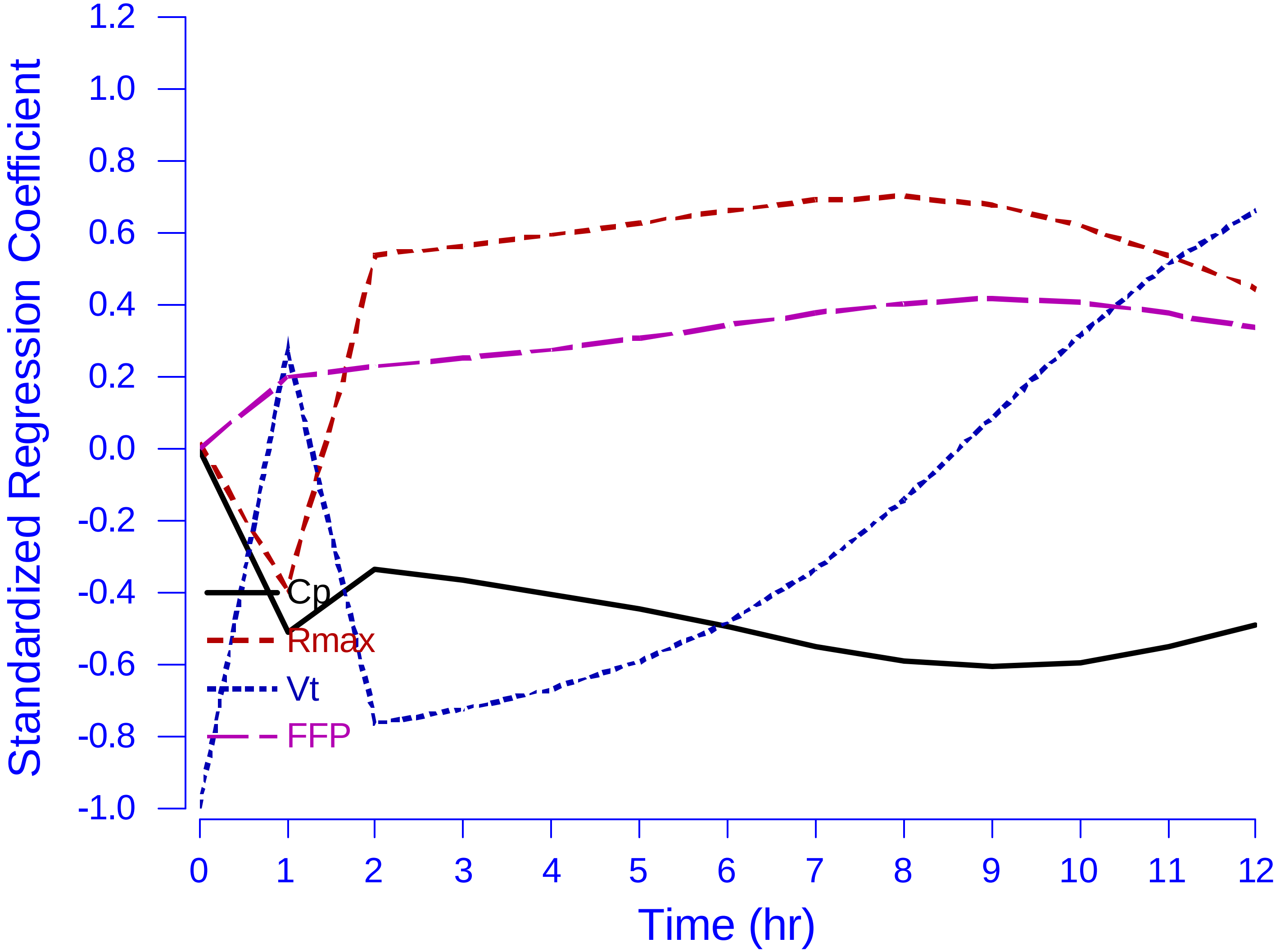
Product

1				
0	1			
0	0	1		
0	0	0	1	
0	0	0	0	1

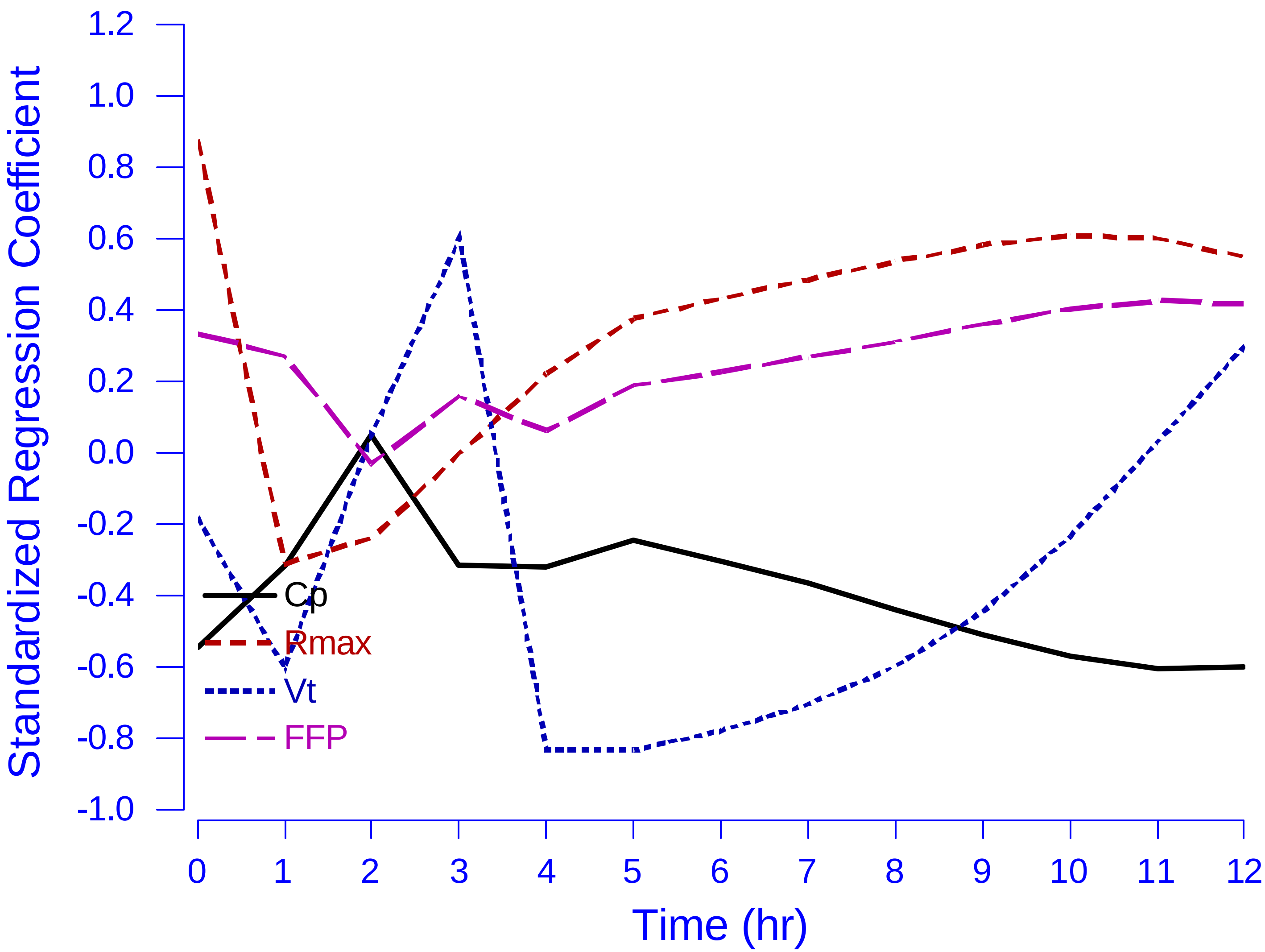
Cat 5 Comparison of Simple, PCC, & SRC at (30, 0) for t=1hr

	Simple Corr	Partial Corr	St. Reg. Coef.
CP	0.187 (3)	-0.871 (3)	-0.212 (3)
Rmax	0.696 (1)	0.989 (1)	0.808 (1)
V_T	0.675 (2)	0.989 (1)	0.696 (2)
FFP	0.006 (4)	0.448 (4)	0.052 (4)

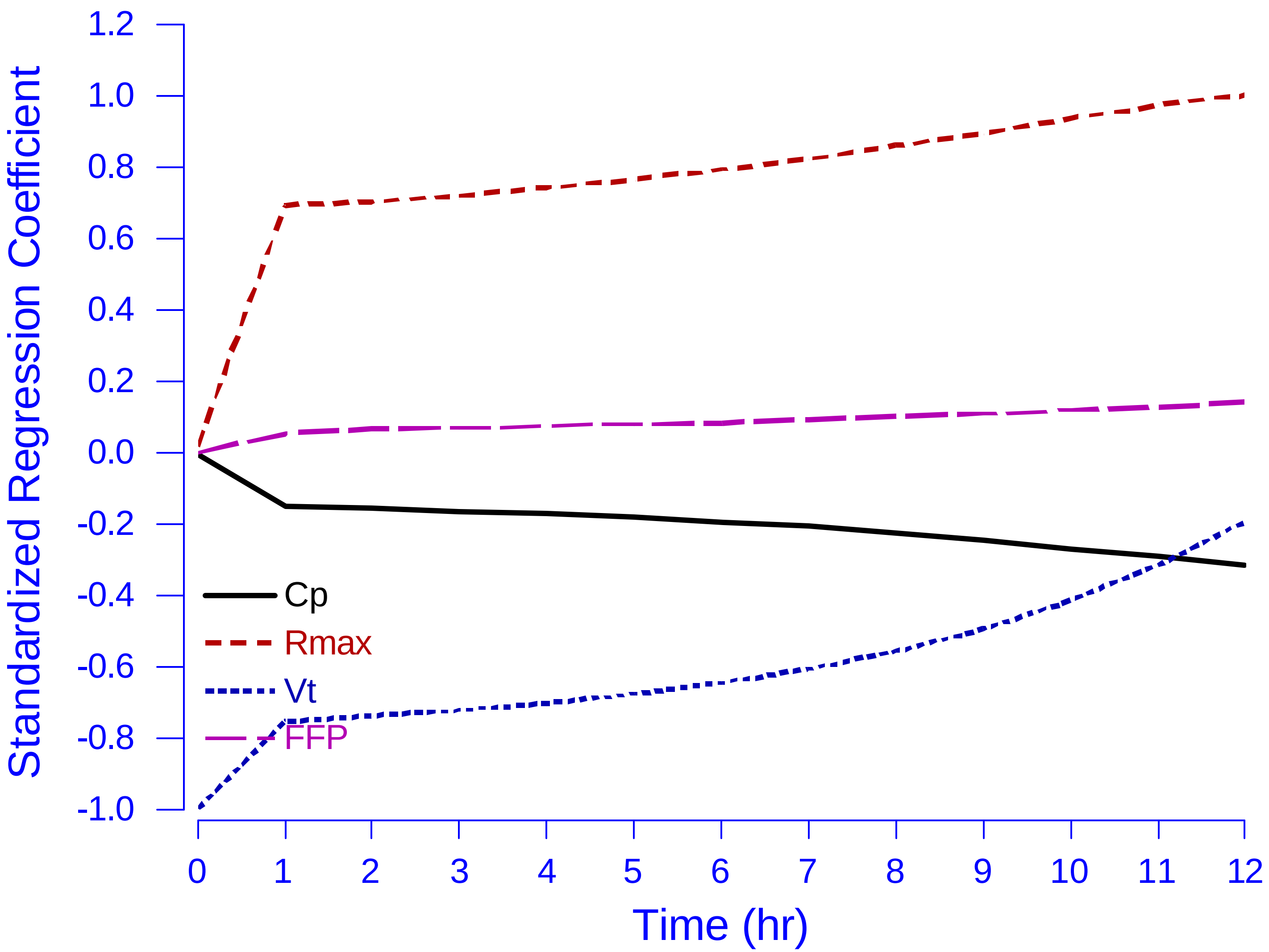
SRC vs Time at (0, 0) for Category 1



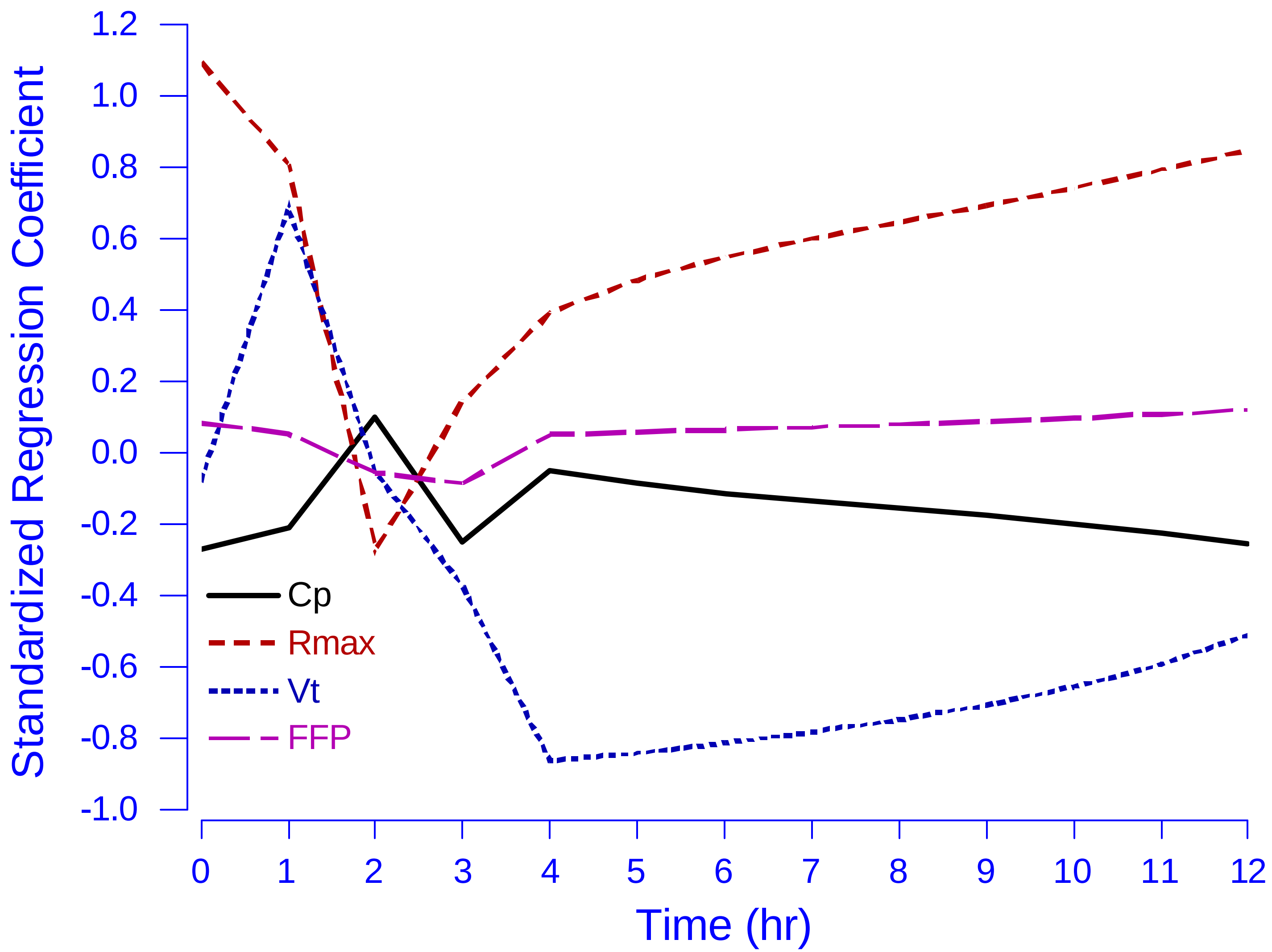
SRC vs Time at (30, 0) for Category 1



SRC vs Time at (0, 0) for Category 5



SRC vs Time at (30, 0) for Category 5



Movies for Categories 1 and 5: SRC

- Track along path of the eye from (0, 0) to (180, 0)
- Track along path 10 miles to the right of the eye from (0, 10) to (180, 10)

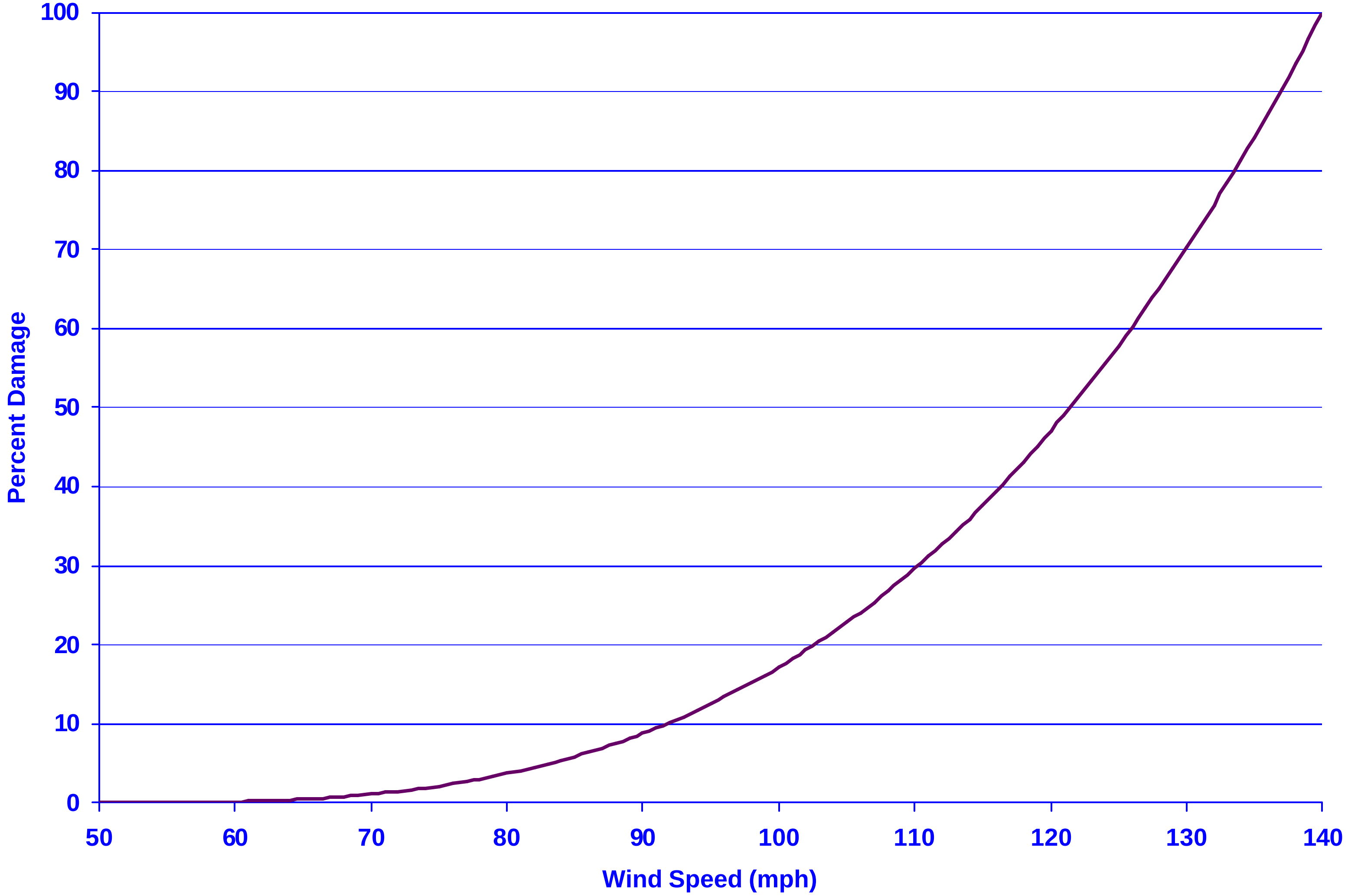
Conversion of V_{Total} to Surrogate Loss Cost

- Assume $100,000 structure at each vertex in the grid with a 1% or $1000 deductible
- Applies to all coordinates with X = 15mi
- Simple demonstration cubic damage function:

$$\%Damage = \left(\frac{V_{Total} - 50}{140 - 50} \right)^3 \quad \text{for } V_{Total} = 50$$

- DISCLAIMER: for demonstration purposes

Graph of Surrogate Damage Function



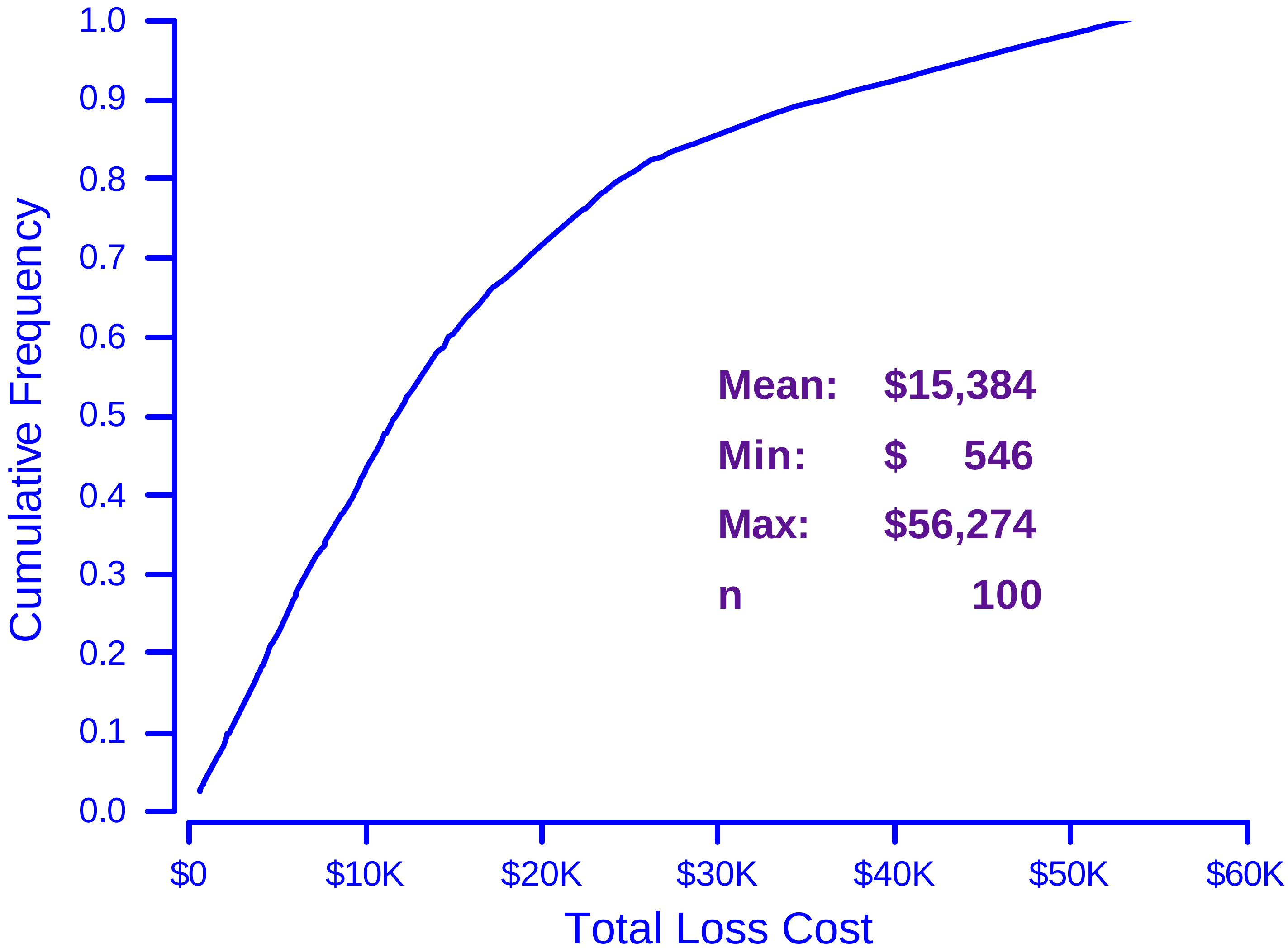
Loss Cost Function

- If %Damage is ≤ 1%, total loss = $0
  This corresponds to $V_{Total} \leq$ 69.39mph

- If %Damage is ≥ 50%, total loss = $99,000
  This corresponds to $V_{Total} \geq$ 121.43mph

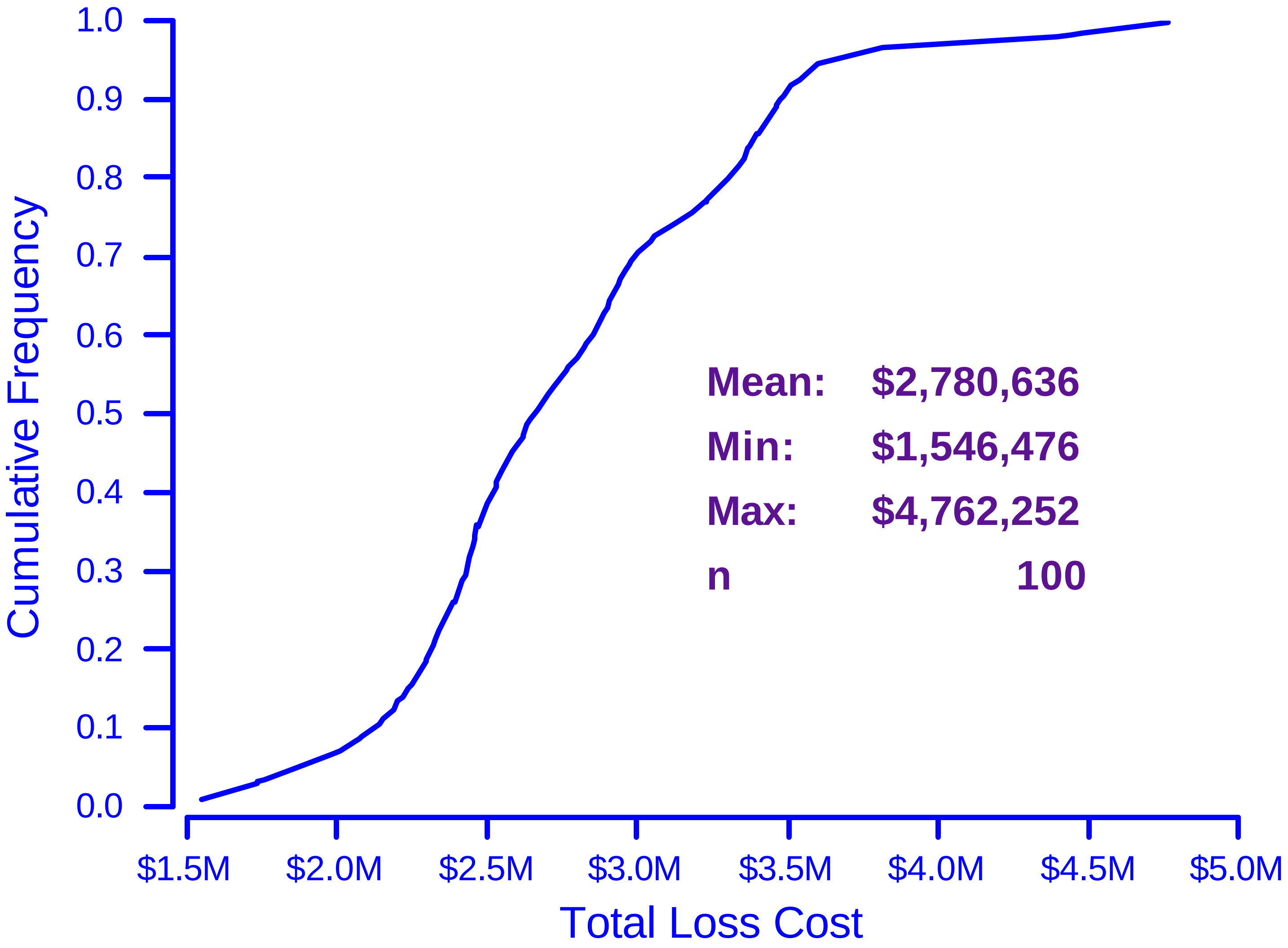
- Otherwise, total loss = %Damage × $100,000 - $1,000

Distribution of Total Loss Cost for a Category 1

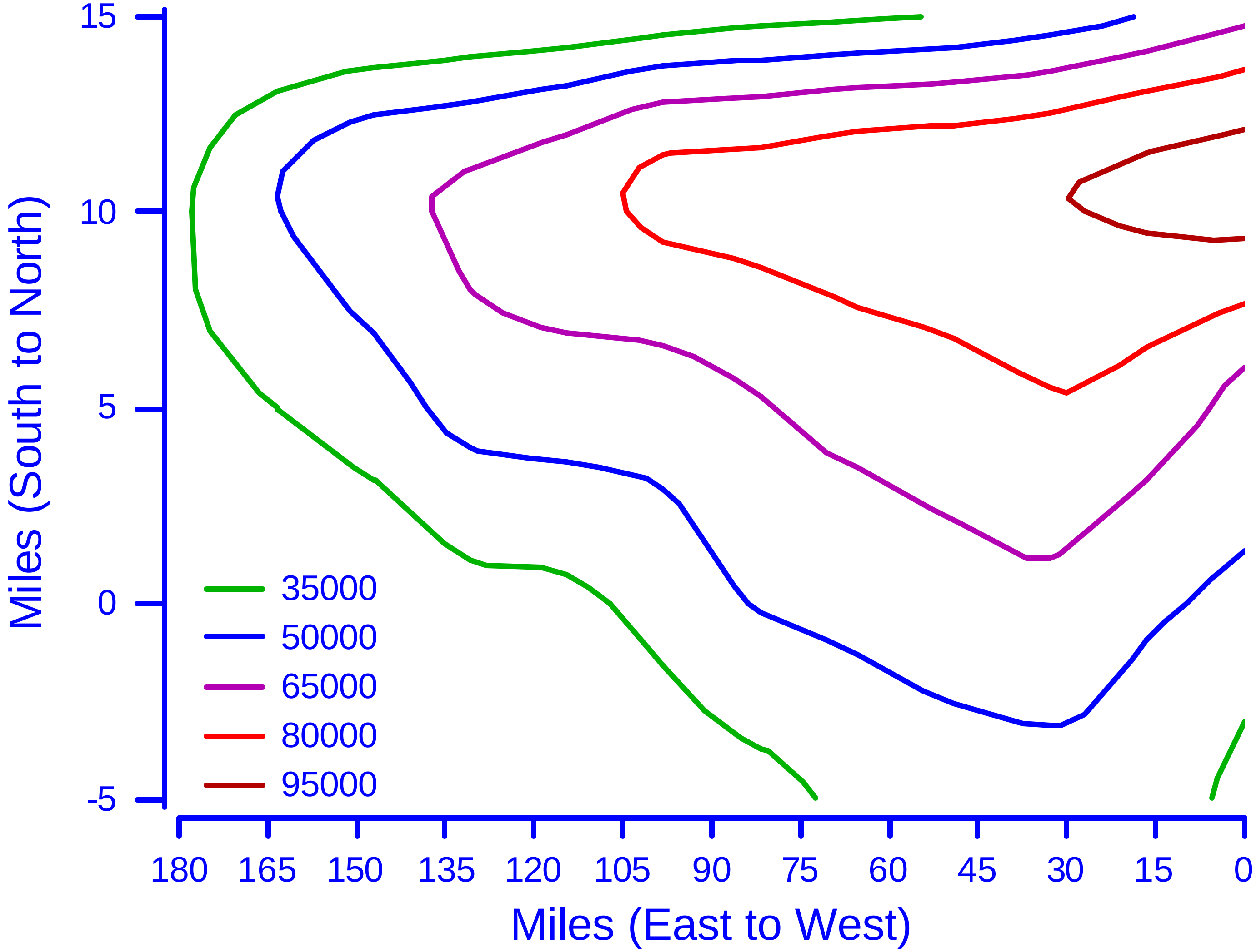


Mean:	**$15,384**
Min:	**$ 546**
Max:	**$56,274**
n	**100**

Distribution of Total Loss Cost for a Category 5



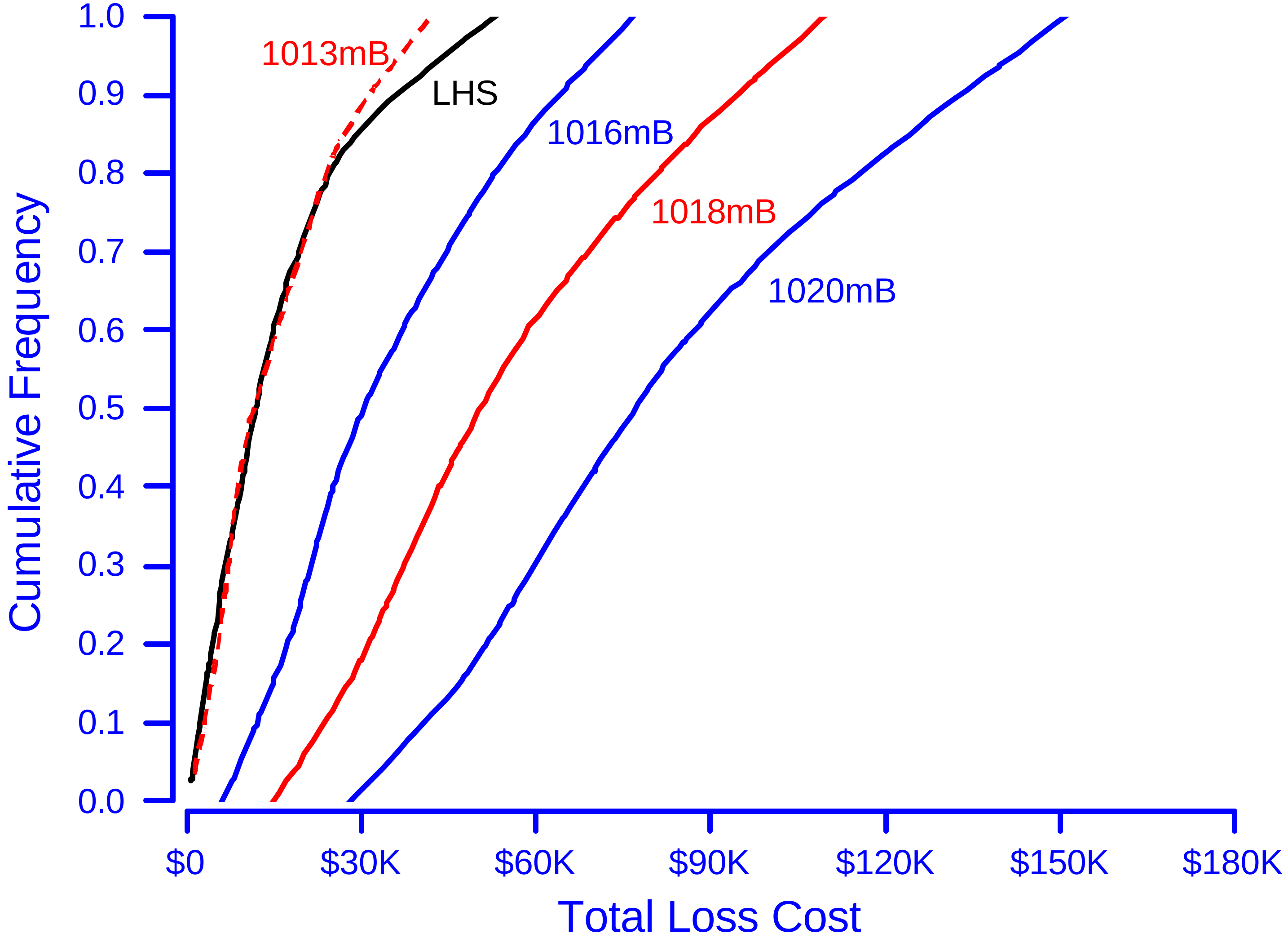
Total Loss Cost for a Category 5 Hurricane



Sensitivity Analysis Results for Loss Cost

		CP	Rmax	V_T	FFP
Category 1	**SRC**	-0.759	-0.026	0.017	0.441
	Rank	1	3	4	2
Category 5	**SRC**	-0.614	0.887	0.076	0.147
	Rank	2	1	4	3

Total Loss Cost for Category 1 as a Function of FFP



Uncertainty Analysis: Which X's Influence Uncertainty

- Well known result in statistics:

$$Var(Y) = E_{X_j}\left[Var(Y \mid X_j)\right] + Var_{X_j}\left[E\left(Y \mid X_j\right)\right] \quad (1)$$

- In words, the variance of Y is equal to the mean of the conditional variance plus the variance of the conditional mean

Uncertainty Analysis: Which X's Influence Uncertainty

- Equation (1) can be rewritten as:

$$Var(Y) - E_{X_j}\left[Var(Y \mid X_j)\right] = Var_{X_j}\left[E\left(Y \mid X_j\right)\right] \qquad (2)$$

- The right-hand side represents the expected reduction in the variance of Y due to ascertaining the value of X_j

Uncertainty Analysis: Which X's Influence Uncertainty

- Dividing both sides of Equation (2) by Var(Y) gives the contribution to the uncertainty in Y attributable to X_j or the expected percentage reduction in Var(Y) due to knowing X_j

$$\frac{Var\left(E\left[Y \mid X_j\right]\right)}{Var(Y)} \times 100\% \qquad (3)$$

Calculating the Expected Percentage Reduction

Step 1.

Estimate the Var(Y) based on *n* = 100 computer runs with the LHS X matrix of sample values

$$X = \begin{bmatrix} X_{1,1} & X_{1,2} & X_{1,3} & X_{1,4} \\ X_{2,1} & X_{2,2} & X_{2,3} & X_{2,4} \\ & \vdots & & \\ X_{n,1} & X_{n,2} & X_{n,3} & X_{n,4} \end{bmatrix}$$

Step 2.

Let X_M represent a vector of sample means

$$X_M = \begin{bmatrix} \overline{X}_1 & \overline{X}_2 & \overline{X}_3 & \overline{X}_4 \end{bmatrix}$$

Calculating the Expected Percentage Reduction

Step 3.

Generate a new matrix X_1^* as

$$X_1^* = \begin{bmatrix} X_{1,1} & \overline{X}_2 & \overline{X}_3 & \overline{X}_4 \\ X_{2,1} & \overline{X}_2 & \overline{X}_3 & \overline{X}_4 \\ & \vdots & & \\ X_{n,1} & \overline{X}_2 & \overline{X}_3 & \overline{X}_4 \end{bmatrix}$$

Step 4.

Run the model using X_1^* and calculate $Var(E[Y|X_1])$, which is the numerator of Equation 3

Calculating the Expected Percentage Reduction

Step 5.

Repeat Steps 3 and 4 for X_2 where the 1st, 3rd, and 4th columns of X are replaced by their respective means and calculate $Var(E[Y|X_2])$

Step 6.

Repeat Steps 3 and 4 for X_3 where the 1st, 2nd, and 3rd columns of X are replaced by their respective means and calculate $Var(E[Y|X_3])$

Calculating the Expected Percentage Reduction

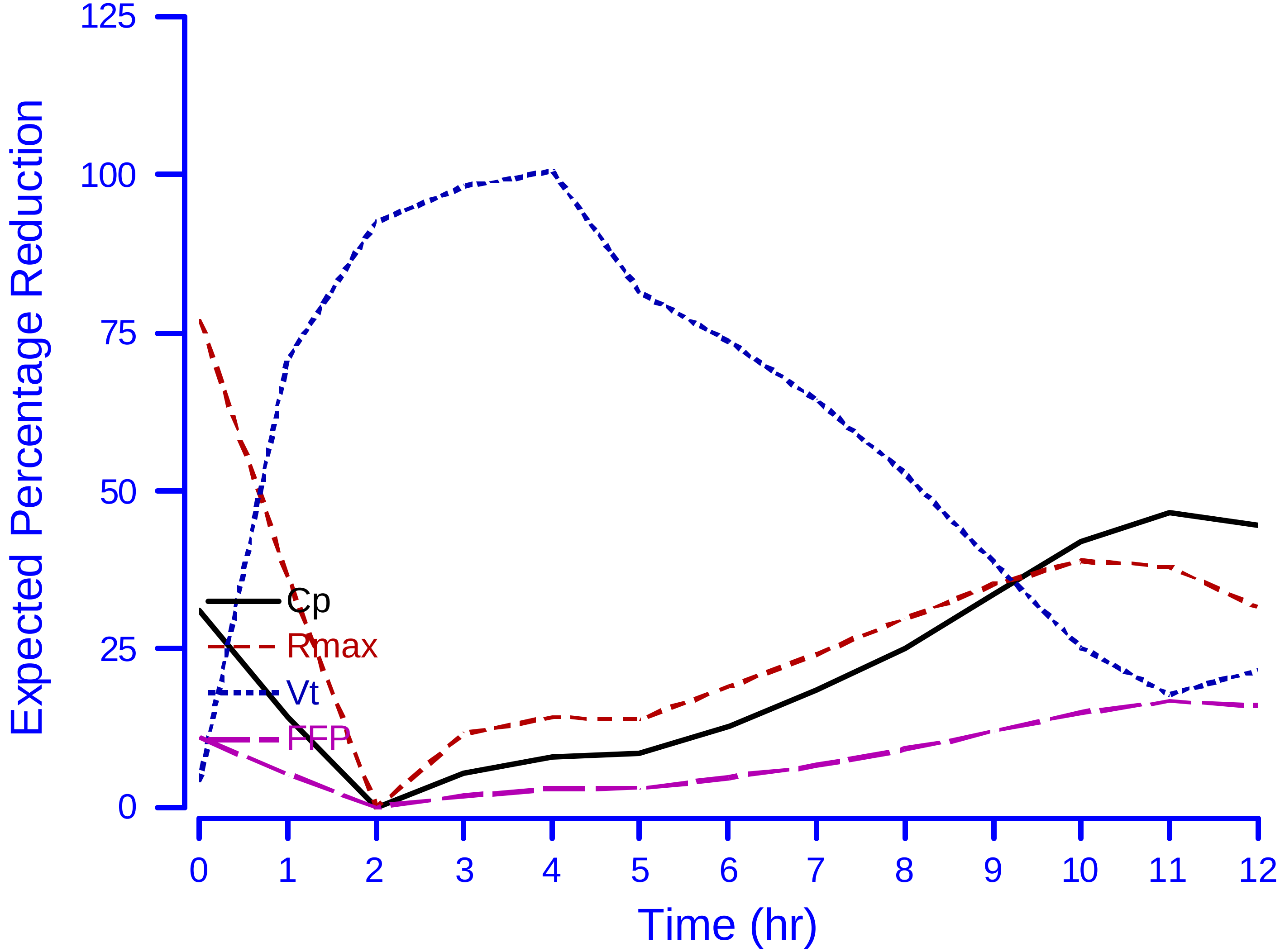
Step 7.

Repeat Steps 3 and 4 for X_4 where the 1st, 2nd, and 3rd columns of X are replaced by their respective means and calculate $Var(E[Y|X_4])$

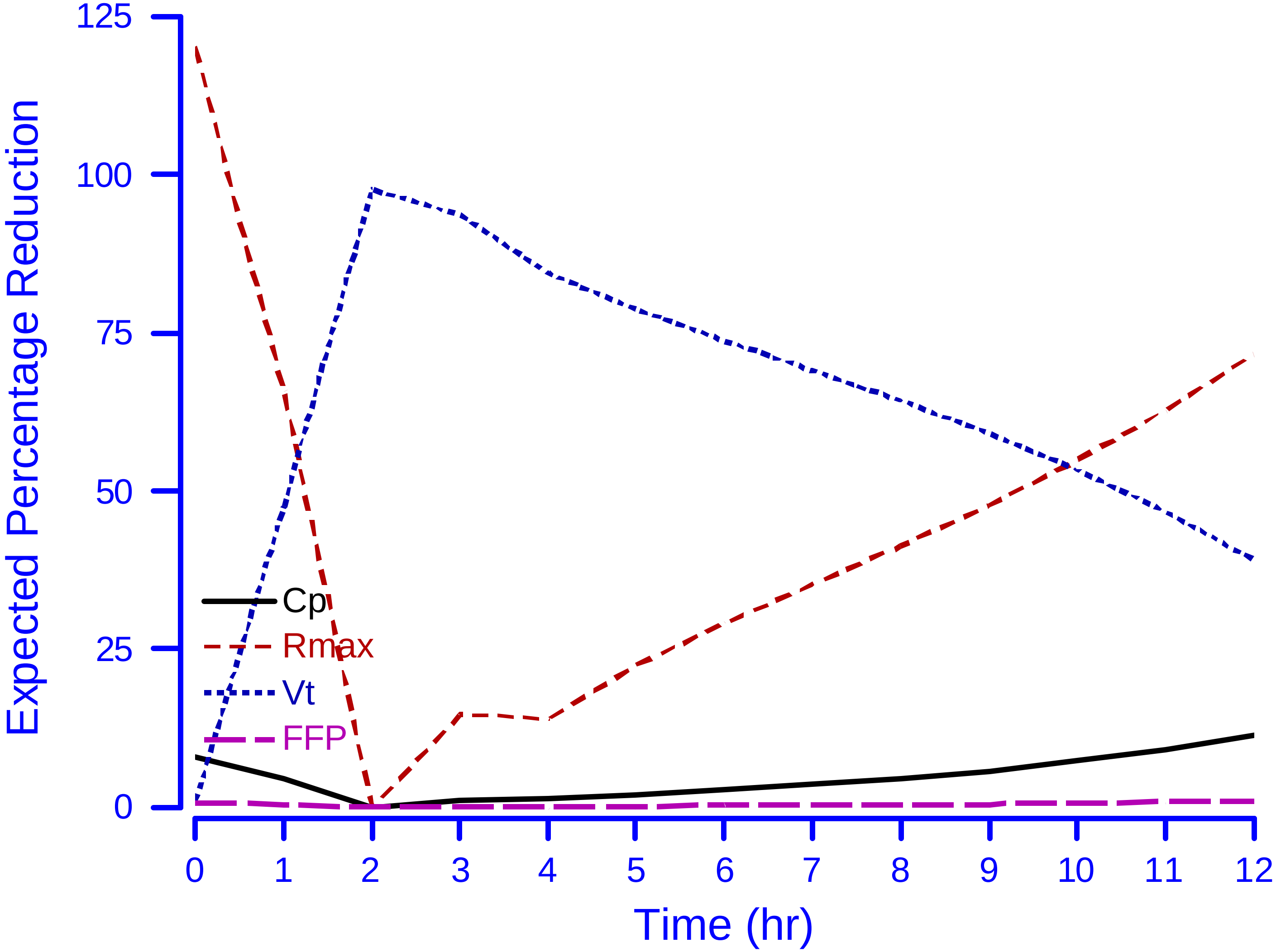
Step 8.

Substitute the estimates in Steps 4-7 into Equation 3 with the estimate of Var(Y) from Step 1 to estimate the expected percentage reductions for X_1 to X_4

Percentage Reduction vs Time at (30, 0) for Category 1

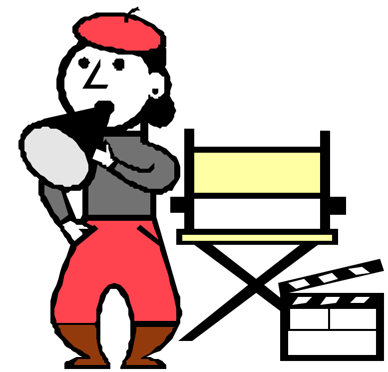


Percentage Reduction vs Time at (30, 0) for Category 5



Movies for Categories 1 and 5: Expected % Reduction

- Track along path of the eye from (0, 0) to (180, 0)
- Track along path 10 miles to the right of the eye from (0, 10) to (180, 10)



Percentage Reductions for Loss Cost

		CP	Rmax	V_T	FFP
Category 1	**Percentage**	92.5%	10.1%	1.8%	28.8%
	Rank	1	3	4	2
Category 5	**Percentage**	8.2%	48.5%	7.0%	0.8%
	Rank	2	1	3	4

Concluding Comments

- The demonstration study has shown that substantial time and spatial variation exists for a given storm on a given track
- Report can serve as a foundation to help the Commission raise the bar with respect to sensitivity and uncertainty analysis
- Pro Team will provide SBA with input files and formats for output files to be the basis for a modified Form F